

# ZAlgs

**Computing nilpotent quotients for finitely  
presented associative  $Z$ -algebras.**

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**Tobias Moede**

**Tobias Moede** Email: [t.moede@tu-braunschweig.de](mailto:t.moede@tu-braunschweig.de)

Homepage: <https://www.tu-braunschweig.de/iaa/personal/moede>

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## Acknowledgements

Comments and suggestions for this package and its documentation are always welcome.

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# Chapter 1

## Introduction

This package contains implementations of the algorithms described in [EM19]. It contains methods to compute the class- $c$  quotient of a finitely presented associative  $Z$ -algebra. As an application this package allows to determine the class- $c$  quotient of the augmentation ideal  $I(G)$  in the integral group ring  $ZG$  for a finitely presented group or a pcp-group  $G$ . This allows to read off the structure of the so-called augmentation quotients  $Q_n(G) = I(G)^n / I(G)^{n+1}$ .

## Chapter 2

# Nilpotent quotients of finitely presented $Z$ -algebras

### 2.1 Computing nilpotent quotients.

The following function allows to compute nilpotent quotients for finitely presented associative  $Z$ -algebras given by a number of generators and relators.

#### 2.1.1 NilpotentQuotientFpZAlgebra

▷ NilpotentQuotientFpZAlgebra( $A$ ,  $c$ ) (operation)

Given a finitely presented associative  $Z$ -algebra, this function computes the class- $c$  quotient.

### 2.2 Computing augmentation quotients of groups.

Let  $I(G)$  denote the augmentation ideal of a group  $G$ , then the following functions calculate the class- $c$  quotient of  $I(G)$  for finitely presented groups and pcp-groups. One can further choose to print the additive structure of the augmentation quotients  $I(G)^i/I(G)^{i+1}$  during computation.

#### 2.2.1 AugmentationQuotientFpGroup

▷ AugmentationQuotientFpGroup( $G$ ,  $c$ ,  $print$ ) (operation)

Given a finitely presented group  $G$ , this function computes the class- $c$  quotient of the augmentation ideal in the integral group ring  $ZG$ . If  $print$  is set to true, then the augmentation quotients are printed during computation.

#### 2.2.2 AugmentationQuotientPcpGroup

▷ AugmentationQuotientPcpGroup( $G$ ,  $c$ ,  $print$ ) (operation)

Given a pcp-group  $G$ , this function computes the class- $c$  quotient of the augmentation ideal in the integral group ring  $ZG$ . If  $print$  is set to true, then the augmentation quotients are printed during computation.

Example

```
gap> H := HeisenbergPcpGroup(1);  
gap> AugmentationQuotientPcpGroup(H, 5, true);  
Q_1 = Z^2  
Q_2 = Z^4  
Q_3 = Z^6  
Q_4 = Z^9  
Q_5 = Z^12
```

# References

- [EM19] B. Eick and T. Moede. A nilpotent quotient algorithm for finitely presented associative  $\mathbb{Z}$ -algebras and its application to integral group rings. *submitted*, 2019. [4](#)

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