ZAlgs

Computing nilpotent quotients for finitely presented associative Z-algebras.

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Comments and suggestions for this package and its documentation are always welcome.

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Chapter 1

Introduction

This package contains implementations of the algorithms described in [EM19]. It contains methods to compute the class-c quotient of a finitely presented associative Z-algebra. As an application this package allows to determine the class-c quotient of the augmentation ideal $I(G)$ in the integral group ring ZG for a finitely presented group or a pcp-group G. This allows to read off the structure of the so-called augmentation quotients $Q_n(G) = I(G)^n/I(G)^{n+1}$.

Chapter 2

Nilpotent quotients of finitely presented Z-algebras

2.1 Computing nilpotent quotients.

The following function allows to compute nilpotent quotients for finitely presented associative Z-algebras given by a number of generators and relators.

2.1.1 NilpotentQuotientFpZAlgebra

▷ `NilpotentQuotientFpZAlgebra(A, c)` (operation)

Given a finitely presented associative Z-algebra, this function computes the class-`c` quotient.

2.2 Computing augmentation quotients of groups.

Let $I(G)$ denote the augmentation ideal of a group G, then the following functions calculate the class-c quotient of $I(G)$ for finitely presented groups and pcp-groups. One can further choose to print the additive structure of the augmentation quotients $I(G)^i/I(G)^{i+1}$ during computation.

2.2.1 AugmentationQuotientFpGroup

▷ `AugmentationQuotientFpGroup(G, c, print)` (operation)

Given a finitely presented group `G`, this function computes the class-`c` quotient of the augmentation ideal in the integral group ring `ZG`. If `print` is set to true, then the augmentation quotients are printed during computation.

2.2.2 AugmentationQuotientPcpGroup

▷ `AugmentationQuotientPcpGroup(G, c, print)` (operation)

Given a pcp-group `G`, this function computes the class-`c` quotient of the augmentation ideal in the integral group ring `ZG`. If `print` is set to true, then the augmentation quotients are printed during computation.

Example

```
gap> H := HeisenbergPcpGroup(1);;
gap> AugmentationQuotientPcpGroup(H, 5, true);;
Q_1 = Z^2
Q_2 = Z^4
Q_3 = Z^6
Q_4 = Z^9
Q_5 = Z^12
```

References

[EM19] B. Eick and T. Moede. A nilpotent quotient algorithm for finitely presented associative Z-algebras and its application to integral group rings. *submitted*, 2019. 4

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