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Introduction to Scientific Computing:
Test

(Time 90 min, Total points 90)

Your Name and Id (Matr.-Number):

Exercise 1: **(25 points)**

Consider the following spring-mass system with unit mass:

$$-\ddot{x} - d\dot{x} - kx = 0 \tag{1}$$

(a) Write as system of order 1 ODEs. (4 points)

(b) Make Eigen decomposition of the system. Write down the general solution of the ODE system. (9 points)

(c) Write down particular solution that matches the initial condition $x_0 = 1$, $\dot{x}_0 = 0$ at $t_0 = 0$. (3 points)

(d) What does the parameter d specify?
For which values of d is the system stable, for which asymtotically stable? (9 points)

Exercise 2: **(25 points)**

(a) Write down Euler-Forward scheme in general, its Butcher scheme and applied to the system from Task 1.a, using $d = 0$ there for brevity. (3 points)

(b) Which kind of numerical integration lies behind this method? (2 points)

(c) Check the zero-stability of the general scheme. (3 points)

(d) Is the numerical solution of the Euler-Forward scheme applied to the system from 1.a, $d = 0$, stable for any h? (14 points)

(e) Check the absolute stability of the given numerical scheme - you may use result from 2.d. (3 points)

Please turn the page

Exercise 3: **(24 points)**

Consider the real function

$$x \longrightarrow f(x) = \tanh(x) := \frac{e^x - e^{-x}}{e^x + e^{-x}} := \frac{\sinh x}{\cosh x}, \tag{2}$$

which is a sigmoid (s-shaped) function. Hint: Making sketches protects you from getting lost. The following is given as help: Using

$$\sinh' x = \cosh x \tag{3}$$

$$\cosh' x = \sinh x, \tag{4}$$

by quotient rule

$$\tanh' x = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = 1 - \tanh^2 x.$$

Both identities may be useful.

(a) Write down the Newton Solver to find the solution x^* of $f(x) = 0$. (4 points)

(b) Where may the starting value x_0 lay such that the Newton iteration will converge? Precisely: Find the interval such that $f(x_n) \longrightarrow 0$ for $n \longrightarrow \infty$ for all x_0 from that interval. Hint: Newtons method induces a difference equation $x_{n+1} = G(x_n)$ and x^* is a fixed point of it. Use Banachs Fixed-Point theorem on it. (20 points)

Exercise 4: **(16 points)**

Given two step Adams-Bashforth method

$$y_{n+2} = y_{n+1} + h\left(\frac{3}{2}f(t_{n+1}, y_{n+1}) - \frac{1}{2}f(t_n, y_n)\right),$$

(a) Prove that it is consistent. (6 points)

(b) Determine its consistency order. (6 points)

(c) Elaborate shortly what is the difference between global and local errors of a numerical scheme. (4 points)