Institute of Scientific Computing
Technical University Braunschweig
Dr. Thilo Moshagen
Dr. Anna Veselovska

Summer Term 2019

Advanced Methods for ODE and DAE: *Assignment 2*

This is to reconsider everything about stability of one-step methods.

Exercise 1: *Due date: 26.4.* **(28 points)**

(a) You learned in ODEI that loss of numerical stability can be seen as overshooting - think of Euler-Forward method applied to $\dot{x} = -x$, $x_0 = 1$ and stepwidth $h = 2.01$.
I intended to design an explicit method that avoids this:

1. The method is based on any explicit RK

$$\mathbf{k}_i = f(t_n + c_i h, \mathbf{x}_n + h\mathbf{A}_{i,\cdot}(\mathbf{k}_j)_{j=1\ldots s}).$$

Remember that equivalently and alternatively, one can calculate not intermediate slopes, but values

$$\mathbf{X}_i = \mathbf{x}_n + h\mathbf{A}_{i,\cdot}(f(t_n + c_i h, \mathbf{X}_j))_{j=1\ldots s}.$$

Even when evaluating $\mathbf{k}_i$, they are evaluated anyway. We store them.

2. In time step n, during calculation of the slopes $\mathbf{k}_i$, each component j of the $\mathbf{k}_i$ is compared against the component $(\mathbf{k}_i)_j$ of the preceding time step $n-1$: If $\|(\mathbf{k}_i)_j - (\mathbf{k}_{i,old})_j\| > 1.1 * \|(\mathbf{k}_{i,old})_j\|$, then introduce a new smaller stepwidth h_j just for this component.

3. Reevaluate $(\mathbf{k})_j$ (the slopes for the solution component j) again using $\mathbf{x}$- values from polynomial interpolation of $\mathbf{X}_i$ to $t_n + c_i h_j$. Test above criterium again.
Evaluate the j-th line of

$$\mathbf{x}_{n+1} = \mathbf{x}_n + h_j \mathbf{b}\mathbf{k}$$

and repeat this with the time interval $[t_n + h_j, t_n + 2h_j)$ and so on until you reach $t_n + h$.

Show that the method solves Dahlquists problem in a stable way. It solves stiff linear problems in a stable way, allowing an efficient stepsize for most component. (14 points)

(b) Is this method unknown because nobody discovered it or because there is a (meaning one or more) problem with it? If there is an example problem for which the method cannot inherit stability to the numerical solution, give it and describe what happens. It is sufficient to sketch the properties of the vector field given by such an RHS well and describe what can happen. (14 points)