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Winter Term 2017
November 24, 2017

Introduction to Scientific Computing
NEWTON METHOD

Exercise 1: **(12 points)**

Let $f(x) = (x - x_*)^m$, i.e. the function has a zero x_* with multiplicity m. Consider a modified Newton's method :

$$x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)} = g(x_k), \qquad 0 \neq m \in \mathbb{N}$$

which could be used to solve the equation $f(x) = 0$.

(a) Compute the derivative of $g(x_k)$. (4 points)

(b) Show that for this function $f(x)$, the modified Newton's method converges quadratically in the neighbourhood of x^* in solving $f(x) = 0$. (4 points)

(c) Show that for the same case the Newton's method converges only linearly in the neighbourhood of x^*. (4 points)

Exercise 2: **(24 points)**

Solve the following nonlinear system

$$x_1 - \frac{1}{10}[1 - x_2 - \sin(x_1 + x_2)] = 0$$

$$x_2 - \frac{1}{10}[2 + x_1 + \cos(x_1 - x_2)] = 0$$

in $\mathbb{R}^2$ by using

(a) Newton method (6 points)

(b) Stationary Newton method (6 points)

(c) Newton method with restarts in which the restart happens every three iterations (6 points)

Compute the solution $x_* = (x_{1*}, x_{2*})$ by starting from the initial point $(0, 0)^T$ and satsfying the convergence criteria

$$\epsilon = \frac{\|x^{(k)} - x^{(k-1)}\|_2}{\|x^{(k-1)}\|_2} \leq 1e - 6$$

Each method has to be written in a form of function

```
function [x,err,solution]=newtonf(func,jfunc,x,miter,tol,other)

% Newton method
%
```

```
% Input
%
%  func - function handle
%  jfunc - Jacobian handle
%  x - initial value
%  miter - maximal number of iterations
%  tol - tolerance
%  other - other arguments
%
% Output:
%   x -  solution in all iterations
%   err - error in all iterations
```

(d) Once the solution is computed, compare the convergence plots (error in logarithmic scale vs. number of iterations) on the same figure. Which method requires the smallest number of iterations? Compare the computational times. Which method is computationally the cheapest? Explain why. (6 points)