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Introduction to Scientific Computing
Test example, 70 Points

Exercise 1: *Solving ODE* **(14 points)**

Solve the ODE

$$\dot{u} = \lambda u, \quad \lambda \ll 0$$
$$u(0) = 1$$

(a) analytically (2 points)

(b) by using (1) Euler forward and (2) Euler backward methods, i.e. write down an approximation formulae of the form $u_{m+1} = p(h\lambda)u_m$, where p is the rational function. (6 points)

(c) For which step-size h applies the relation $u_{m+1} < u_m$ in the item (b) (6 points)

Exercise 2: *Linear difference equation* **(14 points)**

Consider the linear difference equation

$$x_{n+2} = \frac{1}{4}\left(3x_{n+1} + x_n\right)$$

(a) Rewrite it as a first order system. (2 points)

(b) Write down the general solution of the system. (4 points)

(c) Write down the particular solution for $x_0 = (1, 0)^T$ (4 points)

(d) Determine the equilibrium points. Are they stable or not? (4 points)

Exercise 3: *Equilibria of ordinary differential equations* **(14 points)**

Consider the following nonlinear ordinary differential equation:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y + (x^2 - 1)/3 \\ (1 - y^2)/3 \end{pmatrix}.$$

(a) Determine the equilibria points. (6 points)

(b) Are these points stable or not? (8 points)

Exercise 4: *Newton's method* **(14 points)**

Consider the following nonlinear equation:

$$F(x,y) = 0 \quad \text{with} \quad F(x,y) = \begin{pmatrix} x^2 - \sin y \\ x + y \end{pmatrix}$$

(a) Write down the Newton algorithm for this equation. (6 points)

(b) Calculate one step of Newton's algorithm starting with $x_0 = (1,0)^T$. Remember that

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

(8 points)

Exercise 5: *Theory* **(14 points)**

(a) What is the definition of consistency? (6 points)

(b) Derive consistency conditions for linear multistep methods. (8 points)