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Introduction to Scientific Computing
ODEs

Exercise 1: **(12 points)**

Given a function $f(x)$. Derive a formula for the approximation of the definite integral

$$\int_{x_0}^{x_2} f(x)\, dx$$

by interpolating f at the supporting points x_0, $x_1 := (x_0 + x_2)/2$ and x_2 with a polynomial of second order. The polynomial can be easily integrated analytically instead of f. The resulting rule for integration is the so-called *Kepler's barrel rule*.

Exercise 2: **(10 points)**

(a) Determine lower, upper and midpoint estimates of

$$\int_0^\pi \sin t dt$$

by splitting the interval $[0, \pi]$ into 10 uniform subintervals. (6 points)

(b) Determine the absolute and relative errors of these estimates by comparing it to the exact analytical value. (4 points)

Exercise 3: *Newton-Cotes* **(14 points)**

(a) Write a program with which you can determine the weights

$$w_k = \frac{1}{b-a}\int_a^b \prod_{\substack{i=0\\ i\neq k}}^{n} \left(\frac{x - x_i}{x_k - x_i}\right) dx = \int_0^1 \prod_{\substack{i=0\\ i\neq k}}^{n} \left(\frac{n \cdot t - i}{k - i}\right) dt.$$

of the Newton-Cotes formulae

$$Q_n = (b-a)\sum_{k=0}^{n} w_k f(x_k).$$

Give a table of these weights up to $n = 8$. (6 points)

(b) Given $f(x) = e^{2x}$. Determine with the help of the Newton-Cotes formulae the definite integral $\int_{-1/2}^{+1/2} f(x)\, dx$. (4 points)

(c) What happens if you increase the degree of the Polynomial in item (b). (4 points)