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Introduction to Scientific Computing: NUMERICAL INTEGRATION

Due date: 12.1. 2018.

Exercise 1: *The limit to resonance* **(16 points)**

Consider the below spring-mass problem with an excitation term $\sin(\omega\, t)$, as it was to be solved in last homework.

$$\frac{d^2x}{dt^2} + kx = \sin(\omega, t), \qquad x(0) = 0, \left.\frac{dx}{dt}\right|_{t=0} = -\frac{\omega}{\omega^2 + \lambda^2} + \delta$$

(a) Take the general solution of that inhomogeneous ODE, and match it to the initial conditions given. Sketch or plot the solution in an interval that shows the features of interest. (4 points)

(b) Consider the case where $\omega \to \lambda$ for that initial condition. Sketch 2-3 of them, making clear what happens.
What is the limiting case? Find it by considering where above solution goes to. Sketch it. Prove it fulfils ODE. (12 points)

Exercise 2: *From Lecture:* **(12 points)**

Given two ODE:

$$\begin{aligned} \frac{dx}{dt} &= -x, \qquad x(0) = 1 \\ \frac{dx}{dt} &= -2000x, \qquad x(0) = 1 \end{aligned}$$

(a) Write and run a Matlab program to implement explicit Euler method on this ODE with the step size respectively taking values $\frac{1}{500}$, $\frac{1}{1000}$, $\frac{1}{2000}$ and $\frac{1}{4000}$. Give some plots. (8 points)

(b) Compare the results by a log convergence plot and explain it. (4 points)

Exercise 3: **(8 points)**

Derive the weights in the below multistep methods:

(a) Adams-Bashforth formula (3rd degree). (4 points)

(b) Nyström formula (3rd degree). (4 points)