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Introduction to Scientific Computing
ODEs

Exercise 1: **(10 points)**

Let us consider continous analogue of the discrete predator–prey–model. Let $x(t)$ denote the number of prey and $y(t)$ the number of predators at time t. Then we may assume them to evolve according to the system of differential equations

$$\begin{aligned} \dot{x} &= \alpha x - \beta xy, \\ \dot{y} &= \gamma xy - \sigma y, \end{aligned} \tag{1}$$

with the initial conditions $x(0) = x_0$ and $y(0) = y_0$. Here, α is the constant birth rate of prey whereas βy is the mortality of prey depending on the number of predators. γx is the birthrate of predators depending on the number of prey eaten, and σ is the mortality rate of predators.

(a) Compute the equilibria points of this nonlinear system of ODEs. (6 points)

(b) Are these equilibria points stable? Why? (4 points)

Exercise 2: **(16 points)**

Let be given a mass $m = 1$ suspended with a spring of stiffness $K_s = 100$. At time $t_0 = 0$ the mass is realised and allowed to drop.

(a) Model the mass motion $y(t)$ in a form of the second order differential equation. (3 points)

(b) Transform the second order system to the first order. (4 points)

(c) Analytically solve the first order ODE given $y(0) = 0$, $\dot{y}(0) = 0$ and $g = 9.81$. Plot the response. (3 points)

(d) Compute the equilibria points and check their stability. (6 points)

Exercise 3: **(10 points)**

Use Picard's iteration to solve the following ODE:

$$\dot{x} = 3x$$