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Introduction to Scientific Computing
NEWTON METHOD

Exercise 1: *Solution of a Non-linear System of Equations* **(36 points)**

Consider the following system of equations:

$$\begin{aligned} 4.72\sin(2x) - 3.14e^{y} - 0.495 &= 0 \\ 3.61\cos(3x) + \sin(y) - 0.402 &= 0. \end{aligned}$$

For the tolerance $eps = 1\text{e-}10$

(a) implement in Matlab classical Newton method. Use $x_0 = 1.5, y_0 = -16$ as start values, and plot convergence with respect to the solution, residual and derivative on the same graph. What do you notice? Change start values to $x_0 = 15, y_0 = -16$. Perform same computation, and compare convergence of solution with respect to the starting point. What do you observe? Explain results. Measure computation time in both cases by using commands tic and toc. (12 points)

(b) implement in Matlab modified Newton method in which Jacobian is approximated by

1. $\mathbf{M}_k = F'(\mathbf{x}_k)$ if $(k \mod m = 0)$ and $\mathbf{M}_k = \mathbf{M}_{k-1}$ otherwise.
2. $\mathbf{M}_k = \text{diag}(F'(\mathbf{x}_k))$ if $(k \mod m = 0)$ and $\mathbf{M}_k = \mathbf{M}_{k-1}$ otherwise. Use the method to solve the previous nonlinear system by using $m = 1, 2, 5$ and 10 and $x_0 = 1.5, y_0 = -16$.

Plot convergence results for solution on the same graph. What do you observe? Which method converges faster, and which is computationally cheaper? Explain results. Repeat computation for the starting point $x_0 = 15, y_0 = -16$. Why method does not converge for all m's? (12 points)

(c) implement in Matlab the Broyden's method using initial point $x_0 = 1.5, y_0 = -16$. Approximate starting Jacobian by your own choice. Compare results obtained by Broyden's and classical methods. Explain results. (12 points)

Note: for solving linear system of equations use direct solver unless conditions for the conjugate gradient method are satisfied (in this case use pcg command).