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Introduction to Scientific Computing
FIXED POINT ITERATION

Exercise 1: **(10 points)**

Show that if the matrix A is strictly diagonally dominant, then $\|M\|_\infty < 1$ in which M is the iteration matrix of Jacobi method.

Exercise 2: **(18 points)**

Take the matrices from Homework 3, Ex2. d) for the matrix A, and observe the system

$$Ax = b, \quad b = \text{randn(n,1)} \in \mathbb{R}^n, \quad x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ .. \\ 0 \end{pmatrix}$$

(a) Can one obtain the solution of the previous system by Gauss-Seidel and Jacobi methods? Explain why. (4 points)

(b) Predict the absolute error of 100th Gauss-Seidel iteration w.r.t. exact solution without perfoming full iteration process. (4 points)

(c) Give numerical bound for Gauss-Seidel Lipschitz constant. (2 points)

(d) Implement Gauss-Seidel method in Matlab and perform iteration (maximum number of iterations 100). Compute the relative errors in successive iterations w.r.t. different norms and plot them. What do you observe? Stop iteration when you achieve accuracy of 1e-3. (4 points)

(e) Implement conjugate gradient method (maximum number of iterations 100). Compute the relative errors in successive iterations w.r.t. different norms and plot them. What do you observe? Stop iteration when you achieve accuracy of 1e-3. Run iteration again by using preconditioner consisting of diagonal part of the matrix. Plot errors again. Which method gives faster result and which is more accurate? (4 points)

Exercise 3: **(8 points)**

Use Gram-Schmidt orthogonalisation to find the orthogonal basis given vectors $v_1 = (1,1,1)^T, v_2 = (1,1,0), v_3 = (1,0,0)$. Represent each of vectors in obtained basis.