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Introduction to Scientific Computing
FIXED POINT ITERATION

Exercise 1: **(8 points)**

(a) Give an example, with proof, of a function $f : \mathbb{R} \mapsto \mathbb{R}$ that is differentiable but not Lipschitz continuous. (4 points)

(b) Give an example, with proof, of a function $f : \mathbb{R} \mapsto \mathbb{R}$ that is Lipschitz continuous but not differentiable. (4 points)

Exercise 2: **(8 points)**

Prove that a sequence

$$x_{n+1} = (x_n + 10)^{1/4}$$

is Cauchy sequence on $(0, +\infty)$.

Exercise 3: **(8 points)**

For the following equations

$$x = -e^x(x^3 + 1)$$

and

$$\begin{aligned} y &= 3 - x^2 \\ x &= 5 - y^2 \end{aligned}$$

(a) check the Banach fixed point theorem and judge if the fixed point iteration will converge. (4 points)

(b) implement in Matlab the fixed point iteration for both cases, compute the relative error between successive iterations and plot. Comment obtained results. (4 points)

Exercise 4: **(12 points)**

The convergence of the fixed-point iterations is relatively slow and can be optimised as further described. Let x^* be the exact solution of the fixed-point equation $x = f(x)$, then q and x_* can be computed approximately as:

$$x_{k+1} - x_* \approx q(x_k - x_*)$$

$$x_{k+2} - x_* \approx q(x_{k+1} - x_*), \quad k = 1, 2, ...$$

This method is called Aitken's Δ^2-process.

(a) Show that:

$$x_* \approx \frac{x_{k+2}x_k - x_{k+1}^2}{x_{k+2} - 2x_{k+1} + x_k} =: z_k.$$

(4 points)

(b) Compute the value z_0 if the function f is given as $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$. For initial value take $x_0 = 7$. (2 points)

(c) Apply now Aitken's Δ^2-process on the sequence z_k to achieve better performance. Write corresponding MATLAB function which calculates new z and gives back the number of iterations for accuracy 10^{-12}. The function has to give back the results in all iterations. (6 points)