Institute of Scientific Computing
Technische Universität Braunschweig
Dr. Bojana Rosić
MSc Ehsan Adeli

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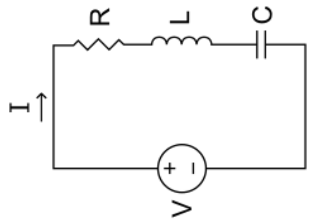
Introduction to Scientific Computing
DIFFERENCE EQUATIONS

Exercise 1: **(12 points)**



Let be given the second order series RLC circuit which can be modelled by a difference equation:

$$(LC + RCh)x_{n+1} + (h^2 - RCh - 2LC)x_n + LCx_{n-1} = Vh^2$$

in which x denotes the current in circuit I. Here, V is the voltage of the power source, R is the resistance of the resistor, L is the inductance of the inductor and C is the capacitance of the capacitor.

(a) Solve the previous difference equation by direct algebraic method given $h = 0.01$, $L = 1$, $R = 8$, $C = 0.25$, $x_0 = 0, x_1 = 15$ and $V = 20$. (3 points)

(b) Transform the previous difference equation into the first order system. (3 points)

(c) Compute stationary point and plot the homogeneous solution (so-called natural response: x as dependence $h \cdot (1 : n)$, where $n = 100/h$) when $R = 8$, $R = 1$ and $R = 0$. Explain why you get different nature of responses? (3 points)

(d) Check stability of the stationary point. (3 points)

Exercise 2: **(12 points)**

Given

$$x_{n+2} - 2x_{n+1} + 2x_n = 3 \tag{1}$$

$$x_{n+3} - 2.5x_{n+2} + 2x_{n+1} - 0.5x_n = 5 \tag{2}$$

$$x_{n+3} - 4.3x_{n+2} + 5.45x_n - 1.275 = 0 \tag{3}$$

(a) Solve the previous difference equations in general form. (6 points)

(b) Compute stationary points and estimate their stability. (6 points)

Exercise 3: **(12 points)**

For the Lorenz system

$$x_{n+1} = x_n + ah(x_n - y_n)$$

$$y_{n+1} = y_n + (rx_n - x_n z_n - y_n)h$$

$$z_{n+1} = z_n + (x_n y_n - bz_n)h$$

written in a matrix form as

$$\mathbf{v}_{n+1} = F(\mathbf{v}_n)$$

where $\mathbf{v} = (x, y, z)^T$ and

$$F(\mathbf{v}_n) = \begin{pmatrix} x_n + ah(x_n - y_n) \\ y_n + (rx_n - x_n z_n - y_n)h \\ z_n + (x_n y_n - bz_n)h \end{pmatrix}$$

(a) find equilibrium points given $a = 10, b = 8/3, r = 28$ and $h = 0.1$. (5 points)

(b) Check stability. (5 points)

(c) Plot the response using iterative approach and Matlab starting from $x_0 = 1.2, y_0 = 1.3, z_0 = 1.6$. (2 points)