Institute of Scientific Computing
Technische Universität Braunschweig
Dr. Bojana Rosić
MSc Ehsan Adeli

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Introduction to Scientific Computing
DIFFERENCE EQUATIONS

Exercise 1: *Algebraic solving* **(14 points)**

Determine order, dimension and degree of the following equations. Classify them with respect to nonlinearity, autonomous properties and homogenity.

(a)

$$x_{n+1} - 3x_n^2 + x_{n-1} = n \tag{1}$$

$$sin(x_{n+1}) - nx_n = 0 \tag{2}$$

$$x_{n+1} - x_n = x_{n+1} \tag{3}$$

$$x_{n+1} - x_n = sin(n) \tag{4}$$

$$\mathbf{x}_{n+1} - \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x}_n = 0 \tag{5}$$

(6 points)

(b) Solve algebraically all difference equations besides the second one. Check obtained solution by iteration. For the initial condition take $x_0 = 1$ if one dimensional equation, and $x_0 = (1,1)^T$ if two-dimensional. (8 points)

Exercise 2: *Matlab* **(22 points)**

Let be given linear difference equation of first order

$$\mathbf{x}_{n+1} = A\mathbf{x}_n, x_0 = (1, ..., 1)^T$$

with $A_{n \times n}$ being a square matrix.

(a) For the matrix $A = BB^T$, $B = \text{randn}(n, n)$ use $n = 10^m$, $m = 1, ..4$, and compute power of $A^k, k = 10, 100, 1000$ if possible. Capture the computation time using commands tic and toc. Gather computation times in a figure with x axis being time and y axis n. Plot lines for different k. What do you notice? (5 points)

(b) Perform same ananlysis only this time use eigenvalue decomposition of the matrix A. Capture computation times and compare them to the results obtained in a). (5 points)

(c) Solve the difference equation in Matlab by taking $m = 3$. Use both power method and eigenvalue decomposition. Evaluate the absolute and relative errors between obtained solutions. (4 points)

(d) Take matrix A from the previous analysis to have same size n as matrices $Problem.A$ given in files cvxbqp1.mat (barrier Hessian matrix by Gould, Hu, and Scott) and bcsstk37.mat (the stiffness matrix by Roger Grimes, Boeing). Plot matrices by using command spy. What do you notice? Perform eigenvalue decomposition of all of three matrices by command $eigs$ by specyfing 100 eigenvalues and capture computation time. Normalise eigenvalues (divide elements in each set by maximum in that set). Plot normalised eigenvalues on one graph for all three matrices. Which matrix requires the largest computation time, and which has the fastest decaying eigenvalues? What it means when eigenvalues decay? Can we use this information to simplify the process of solving our linear difference equation? Explain how without solving the equation. (8 points)