Institute of Scientific Computing
Technische Universität Braunschweig
Dr. Bojana Rosić
MSc Ehsan Adeli

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Introduction to Scientific Computing
LINEAR ALGEBRA

Exercise 1: *Basis of vector space* **(10 points)**

(a) Can vectors $(1, 1, 0, 0), (0, 1, 1, 0), (0, 0, 1, 1)$ and $(1, 0, 0, 1)$ be a basis in vector space defined by tuples (x_1, x_2, x_3, x_4) taking real values? Prove. (6 points)

(b) Can vectors $(2, 1)$ and $(1, -4)$ be a basis in $\mathbb{R}^2$? If yes, then represent the vector $(0, 9)$ in specified basis. Compare the last representation with the one given in basis $(1, 0)$ and $(0, 1)$. What is the difference between them? Which one is more suitable for numerical practice? (4 points)

Exercise 2: *Subspace* **(8 points)**

(a) Prove that the span of vectors $(1, -1, 0), (0, 1, -1)$ coincides with the subspace of $\mathbb{R}^3$ consisting of all vectors (a, b, c) with $a + b + c = 0$. (8 points)

Exercise 3: *Vector norm* **(6 points)**

Take functions $f(t) = t^2 - 1$ and $g(t) = t + 1$ in real argument t. Compute absolute error $\varepsilon_a^I(t)$ between them for each value of $t \in [-1, 1]$. Plot both of functions on this interval, as well as error. Now compute absolute error ε_a^{II} on whole interval by taking norms $\| \cdot \|_{1,2,\infty}$. Can you explain relation between $\varepsilon_a^I(t)$ and ε_a^{II}?

Exercise 4: *Matrix norm* **(12 points)**

a) Prove

$$\|x\|_\infty \le \|x\|_2 \le \|x\|_1 \le \sqrt{n}\, \|x\|_2 \le n\, \|x\|_\infty.$$

b) Prove

$$\frac{1}{d}\|A\|_\infty \le \frac{1}{\sqrt{d}}\|A\|_2 \le \|A\|_1 \le \sqrt{d}\, \|A\|_2 \le d\, \|A\|_\infty$$

with $A \in \mathbb{R}^{d,d}$