Introduction to Scientific Computing
CONDITION NUMBER

Exercise 1: **(12 points)**

(a) (**paper**) Evaluate 1,2, infinity and Frobenius norm of the following matrix

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

Check obtained results with the Matlab built-in functions. (4 points)

(b) (**Matlab**) Let be given the linear system of equations

$$Ax = b$$

in which $b = (1 \quad 1)^T$. Perturb matrix A for 1% and evaluate solution. Repeat the process but this time by perturbing the right hand side for 1%. Compute the relative errors between the non-perturbed and pertrubed solutions. What do you observe? (4 points)

(c) (**paper**) Check if the system is well-conditioned. (4 points)

Exercise 2: **(12 points)**

(a) (**paper+Matlab**) Given the function

$$f(x) = \sqrt{x+1} - \sqrt{x}$$

compute the condition number and check if the function is well conditioned. Support your explanation by a plot. (6 points)

(b) (**paper+Matlab**) For which argument x the function

$$f(x) = 2x^2 + x - 1$$

is well-conditioned? Support your explanation by a plot. (6 points)

Exercise 3: **(12 points)**

(a) (**Matlab**) Let $A_{n\times n}$ be a square matrix with 1's along the diagonal, -1 above the diagonal and 0's below the diagonal. Show that, as n increases, the determinant of A is always 1, but the condition number increases exponentially with n (under the 1, 2 or infinity norms). (6 points)

(b) (**Matlab**) Show that, for increasing n, the diagonal matrix $A_{n\times n}$ with elements on diagonal equal to 0.1, has a determinant that goes to zero (as n goes to infinity), but the condition number is always 1 (under the 1, 2 or infinity norms). (6 points)