Institute of Scientific Computing
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Introduction to Scientific Computing

Homework 11
Jordan normal form

Exercise 1: *Jordan normal form in solving difference equations* **(18 points)**

Given a difference equations system

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n,$$

with

$$\mathbf{A} = \begin{pmatrix} -3 & -6 & 6 \\ 1 & 0 & 6 \\ 0 & -1 & 4 \end{pmatrix},$$

(a) Compute the eigenvalues and corresponding (generalized) eigenvectors of $\mathbf{A}$, state the algebraic multiplicity and geometric multiplicity(number of corresponding eigenvectors) of the eigenvalues. (10 points)

(b) Write out the Jordan matrix $\mathbf{J}$, and the matrix $\mathbf{M}$, so that $\mathbf{A} = \mathbf{M}\,\mathbf{J}\,\mathbf{M}^{-1}$. (4 points)

(c) Write out the general solution of the difference equations system. (4 points)

Exercise 2: *Jordan normal form in solving ODE system* **(18 points)**

Given an ODE system

$$\frac{d\mathbf{u}}{dt} = \mathbf{A}\mathbf{u},$$

with

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix},$$

(a)
Compute the eigenvalues and corresponding (generalized) eigenvectors of $\mathbf{A}$. (10 points)

(b) Write out the Jordan matrix $\mathbf{J}$, and the matrix $\mathbf{M}$, so that $\mathbf{A} = \mathbf{M}\,\mathbf{J}\,\mathbf{M}^{-1}$. (4 points)

(c) Write out the general analytical solution of the ODE system (Hint: in case of deriving from an analogy of difference equations, take $\Delta t = 0$ in this analytical solution). (4 points)