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Introduction to Scientific Computing
Homework 10

Exercise 1: *Stability* **(16 points)**

Given an ODE

$$\frac{dx}{dt} = \lambda x,$$

with $\lambda < 0$,

(a) Write out the 3rd degree Adams-Bashforth scheme on solving this ODE. (2 points)

(b) Derive the stability criteria in time step size. (6 points)

(c) Write and run a Matlab program solving the ODE with the 3rd degree Adams-Bashforth scheme, taking $\lambda = -10$ and $x(0) = 1$ (the other two initial values of x can be taken from analytical solution of the ODE). Test the stability criteria with convergent and divergent examples. (8 points)

Exercise 2: *Accuracy* **(4 points)**

Consider solving the same ODE as in the first exercise,

(a) If we use backward Euler's scheme and trapezoidal scheme, which one would be more accurate? why? (4 points)

Exercise 3: *ODE system* **(16 points)**

Given an ODE system

$$\frac{d\mathbf{u}}{dt} = \mathbf{A}\mathbf{u},$$

with

$$\mathbf{A} = \begin{pmatrix} -3 & 1 \\ 0 & 100 \end{pmatrix} \text{ and } \mathbf{u}(0) = [0, 1]^\top,$$

Let $v_1 = [1, 0]^\top$, $v_2 = [-1/97, 1]^\top$, $C_1 = 1/97$ and $C_2 = 1$, the analytical solution is:

$$\mathbf{u}(t) = C_1 e^{-3t} v_1 + C_2 e^{-100t} v_2,$$

(a) Write and run a Matlab program to solve the ODE system with forward and backward Euler's method. (8 points)

(b) By your observation, what is the value of the stability criteria for the forward Euler's method on this system? (use the analytical solution to measure errors) (2 points)

(c) Explain why the stability criteria takes that value. (6 points)