**Institute of Scientific Computing**
**Technische Universität Braunschweig**
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# Introduction to Scientific Computing
## *Homework 8*

**Exercise 1:** *Solution of homogeneous equation* **(14 points)**

Consider the homogeneous initial value problem

$$\frac{d^2x}{dt^2} = -kx, \qquad x(0) = x_0, \left.\frac{dx}{dt}\right|_{t=0} = 0$$

(a) Transform it to a first order system . (4 points)

(b) Calculate eigenvalues $\lambda_i$ and eigenvectors $v_i$. Find the basis of the solutions. (4 points)

(c) What would an engineer call the $\lambda_i$? (2 points)

(d) Find the solution by matching the initial values. Express the result in trigonometric functions. (4 points)

**Exercise 2:** *Solution of inhomogeneous equation* **(22 points)**

Solve the inhomogeneous problem with a term $\sin(\omega_2 t)$ representing external force

$$\frac{d^2x}{dt^2} = -kx + \sin(\omega_2 t), \qquad x(0) = x_0, \left.\frac{dx}{dt}\right|_{t=0} = 0$$

(a) Calculate a solution without respect to initial conditions by the method of undertermined coefficients, i.e., taking the ansatz $x = a\cos(\omega_2 t) + b\sin(\omega_2 t)$. (3 points)

(b) Calculate a solutions by the method "variation of constants" of the corresponding first order system. (9 points)

(c) Matching the initial condition. (4 points)

(d) What happens when $\omega_2 \longrightarrow \lambda_i$ for one $\lambda_i$, what if $\omega_2 = \lambda_i$? Make good sketches. (4 points)

(e) What does this mean in an engineering sense? Try to give examples. (2 points)