Technische Universität Braunschweig

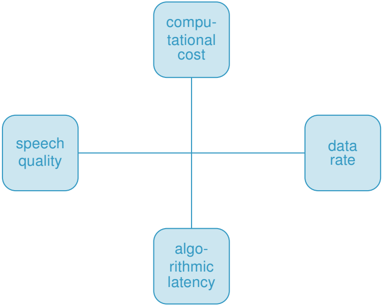
technicolor

Sparse Reconstruction of Quantized Speech Signals

Christoph Brauer, Timo Gerkmann and Dirk Lorenz

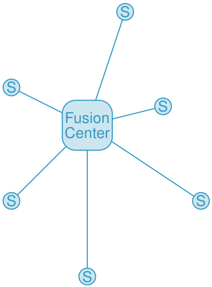
Joint Annual Meeting of GAMM and DMV, March 10, 2016

Speech Coding



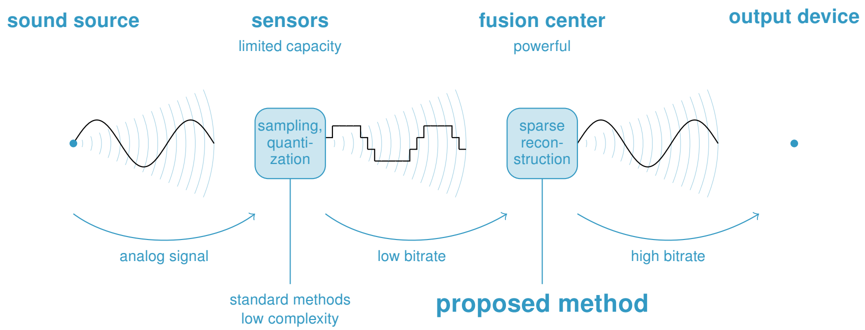
- encoding of analog signals
- decoding of encoded signals
- goal: find a good trade-off between different target values
- requirements depend on the regarded application

Wireless Acoustic Sensor Networks



- *“a next-generation technology for audio acquisition and processing”*
- encoding by distributed, small, cheap sensors
- decoding by a powerful central processor
- applications include hearing aids, hands-free telephony, acoustic monitoring and ambient intelligence

Wireless Acoustic Sensor Networks



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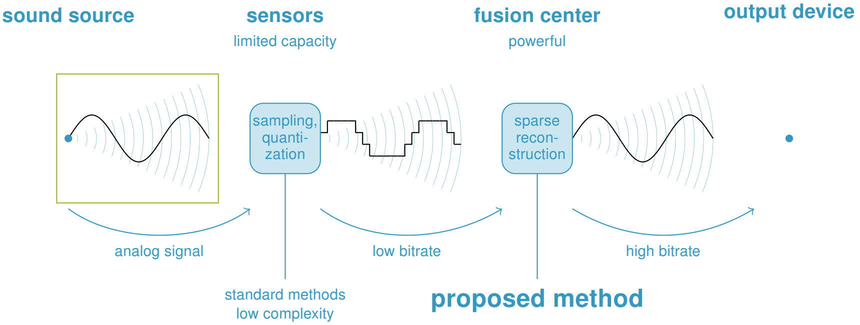

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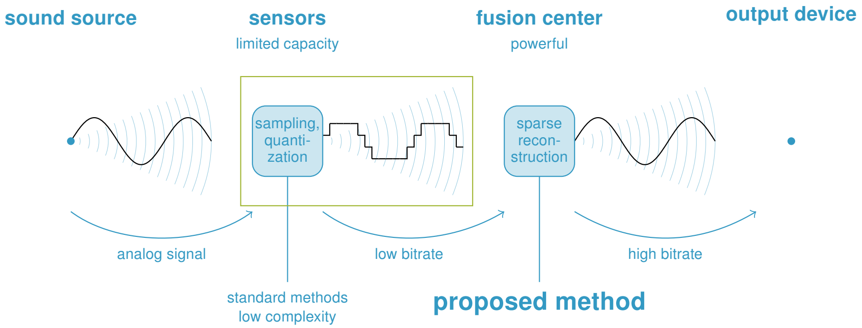

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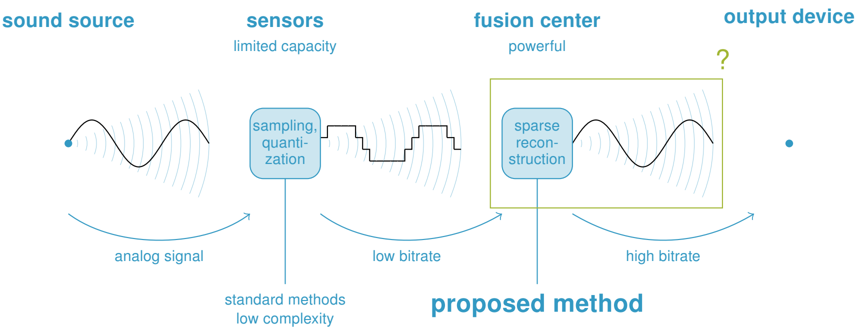
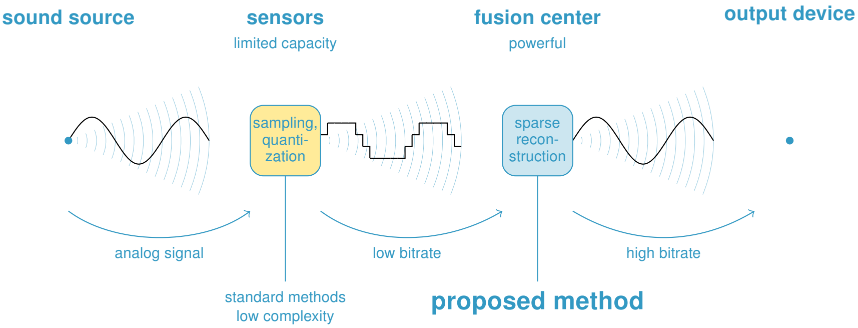


Outline

Encoding


sound source
sensors
limited capacity
fusion center
powerful
output device
sampling, quanti- zation
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analog signal
low bitrate
high bitrate
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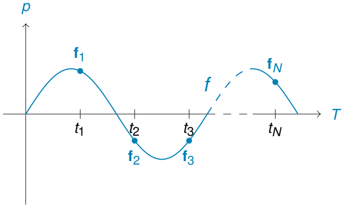
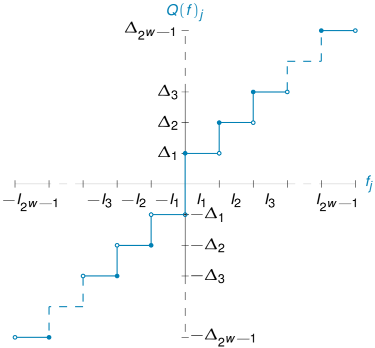
proposed method

Sampling



- $f : T \to (-1, 1)$ analog signal
- $t_j \in T$ equidistant points
- $\mathbf{f} \in (-1, 1)^N$ with $\mathbf{f}_j = f(t_j)$ sampled signal

Quantization

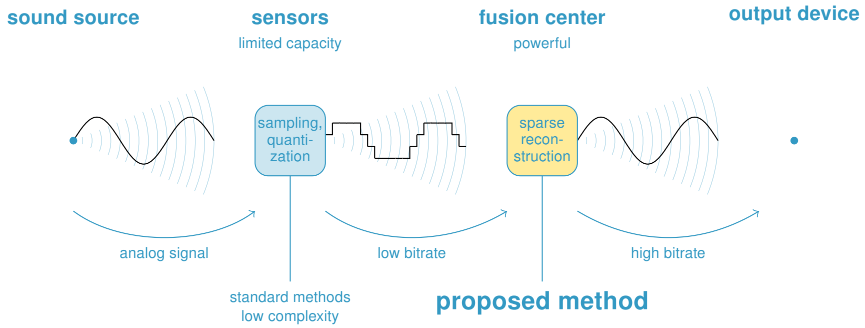


- $\mathbf{f} \in (-1, 1)^N$ speech signal
- $Q : (-1, 1)^N \to (-1, 1)^N$
- $w \in \mathbb{N}$ word length

Quantization Function

$$Q(\mathbf{f})_j = \operatorname{sign}^+(\mathbf{f}_j)\Delta_l \quad \text{if} \quad |\mathbf{f}_j| \in I_l$$

Decoding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General Approach

- $Q(\mathbf{f})$ is given
- seek to find $\mathbf{x}$ that approximates $\mathbf{f}$

Priors

1. $Q(\mathbf{x}) = Q(\mathbf{f})$
2. $\mathbf{x} = \Psi\mathbf{a}$ for full-rank matrix $\Psi \in \mathbb{R}^{N \times N}$ and sparse $\mathbf{a}$

General Approach

Optimization Problem

$$\min_{\mathbf{a}\in\mathbb{R}^N} \|\mathbf{a}\|_1 \quad \text{s.t. } Q(\Psi\mathbf{a}) = Q(\mathbf{f})$$

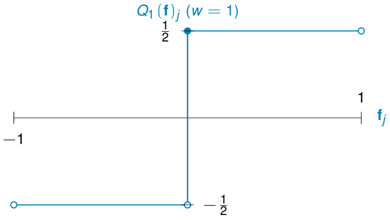
- constraint has a linear reformulation

Relaxed Optimization Problem

$$\min_{\mathbf{a}\in\mathbb{R}^N} \|\mathbf{a}\|_1 \quad \text{s.t. } \forall j : (\Psi\mathbf{a})_j \in \overline{I_{Q(\mathbf{f})_j}}$$

- if Q is given, we need the intervals $\overline{I_{Q(\mathbf{f})_j}}$

Uniform Quantization



- w is given
- $\Delta = 2^{-w+1}$
- $I_l = [(l-1)\Delta, l\Delta)$
- $\Delta_l = (l - \frac{1}{2})\Delta$
- $\overline{I_l} = \{|x - \Delta_l| \leqslant \frac{\Delta}{2}\}$
- $\overline{I_{Q_\Delta(\mathbf{f})_j}} = \{|x - Q_\Delta(\mathbf{f})_j| \leqslant \frac{\Delta}{2}\}$

Optimization Problem

$$\min_{\mathbf{a}\in\mathbb{R}^N} \|\mathbf{a}\|_1 \quad \text{s.t. } \|\Psi\mathbf{a} - Q_\Delta(\mathbf{f})\|_\infty \leqslant \frac{\Delta}{2}$$

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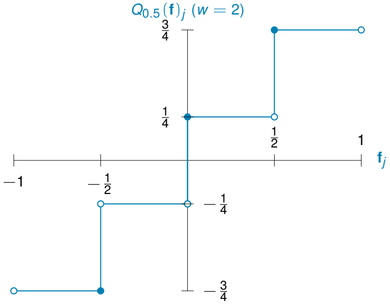

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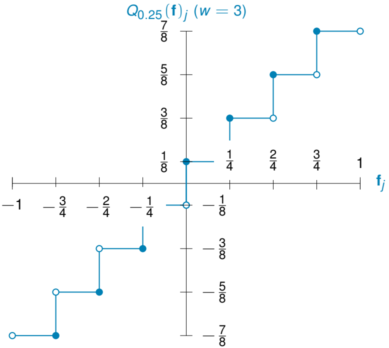

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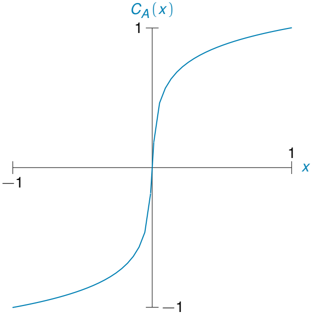
Non-Uniform Quantization

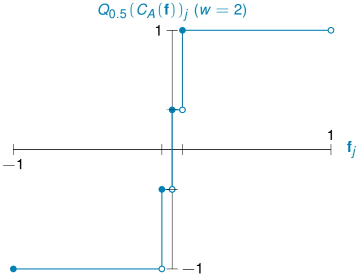


- Δ and Δ_I as before
- $C_A : (-1, 1) \to (-1, 1)$ odd, continuous and strictly monotonically increasing
- $Q_A(\mathbf{f}) = Q_\Delta(C_A(\mathbf{f}))$

A-law

$$C_A(x) = \begin{cases} \operatorname{sign}(x)\frac{1+\ln A|x|}{1+\ln A} & \text{, if } \quad |x| \geqslant \frac{1}{A} \\ \operatorname{sign}(x)\frac{A|x|}{1+\ln A} & \text{, else} \end{cases}$$

Non-Uniform Quantization

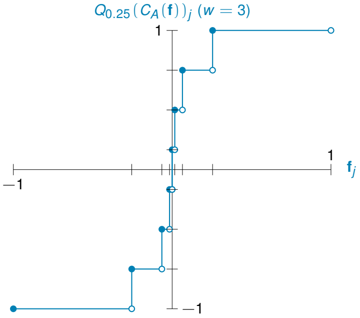


$$
\begin{aligned}
& Q_\Delta(C_A(\mathbf{f}_j)) = \Delta_l \\
\Leftrightarrow & C_A(\mathbf{f}_j) \in [\Delta_l - \tfrac{\Delta}{2}, \Delta_l + \tfrac{\Delta}{2}) \\
\Leftrightarrow & \mathbf{f}_j \in \underbrace{[C_A^{-1}(\Delta_l - \tfrac{\Delta}{2}), C_A^{-1}(\Delta_l + \tfrac{\Delta}{2}))}_{=I_{\Delta_l}}
\end{aligned}
$$

New Interval

$$\overline{I_{Q_A(\mathbf{f})_j}} = [C_A^{-1}(Q_A(\mathbf{f})_j - \tfrac{\Delta}{2}), C_A^{-1}(Q_A(\mathbf{f})_j + \tfrac{\Delta}{2})]$$

Non-Uniform Quantization



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$$

Non-Uniform Quantization

- $\alpha := C_A^{-1}(Q_A(\mathbf{f}) - \frac{\Delta}{2})$
- $\beta := C_A^{-1}(Q_A(\mathbf{f}) + \frac{\Delta}{2})$

New Optimization Problem

$$\min_{\mathbf{a}\in\mathbb{R}^N} \|\mathbf{a}\|_1 \quad \text{s.t. } \alpha \leqslant \Psi\mathbf{a} \leqslant \beta$$

Algorithm

- $\Omega_1 = \{\|\mathbf{x} - Q_\Delta(\mathbf{f})\| \leqslant \frac{\Delta}{2}\}$
- $\Omega_2 = \{\alpha \leqslant \mathbf{x} \leqslant \beta\}$
- Writing $\Omega = \Omega_1$ and $\Omega = \Omega_2$, respectively, we can reformulate above problems as saddle point problems:

$$\min_{\mathbf{a}\in\mathbb{R}^N} \|\mathbf{a}\|_1 + I_\Omega(\Psi\mathbf{a}) = \min_{\mathbf{a}\in\mathbb{R}^N} \max_{\mathbf{y}\in\mathbb{R}^N} \|\mathbf{a}\|_1 + \langle \mathbf{y}, \Psi\mathbf{a}\rangle + I_\Omega^*(\mathbf{y})$$

Algorithm

Chambolle-Pock-Iteration

1. $\mathbf{a}^{i+1} = \operatorname{prox}_{\tau\|\cdot\|_1}(\mathbf{a}^i - \tau\Psi^\top \mathbf{y}^i)$
2. $\overline{\mathbf{a}}^{i+1} = 2\mathbf{a}^{i+1} - \mathbf{a}^i$
3. $\mathbf{y}^{i+1} = \operatorname{prox}_{\sigma I_\Omega^*}(\mathbf{y}^i + \sigma\Psi\overline{\mathbf{a}}^{i+1})$

- *Proximal operator* for convex $F : \mathbb{R}^N \to \mathbb{R}$:

$$\operatorname{prox}_{\lambda F}(\mathbf{x}) := \operatorname*{argmin}_{\mathbf{y}\in\mathbb{R}^n} \lambda F(\mathbf{y}) + \frac{1}{2}\|\mathbf{x} - \mathbf{y}\|_2^2$$

- Has closed-form for $\tau\|\cdot\|_1$ and σI_Ω^*:

Algorithm

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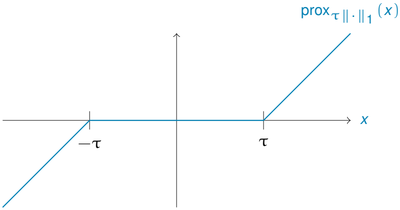

- $\operatorname{prox}_{\tau\|\cdot\|_1}(\mathbf{a}) = \mathbf{a} - \mathcal{P}_{\{\|\cdot\|_\infty \leqslant \tau\}}(\mathbf{a})$
- $\operatorname{prox}_{\sigma l_{\Omega_1}^*}(\mathbf{y}) = \operatorname{prox}_{\frac{\sigma\Delta}{2}\|\cdot\|_1}(\mathbf{y} - \sigma Q_\Delta(\mathbf{f}))$
- $\operatorname{prox}_{\sigma l_{\Omega_2}^*}(\mathbf{y}) = \mathbf{y} - \mathcal{P}_{\{\sigma\alpha \leqslant \cdot \leqslant \sigma\beta\}}(\mathbf{y})$

Algorithm

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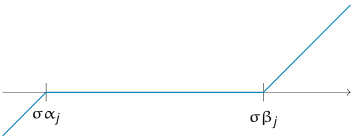

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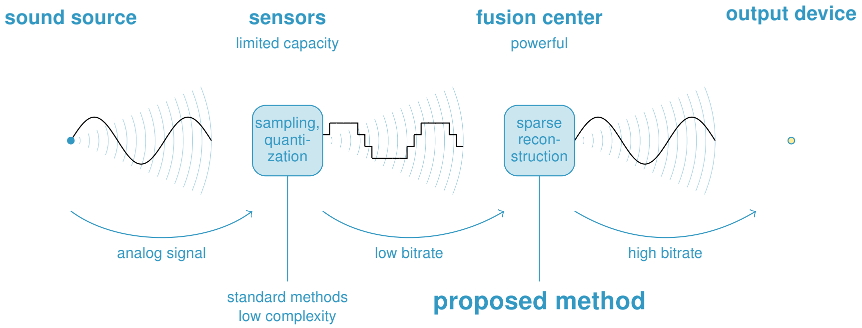
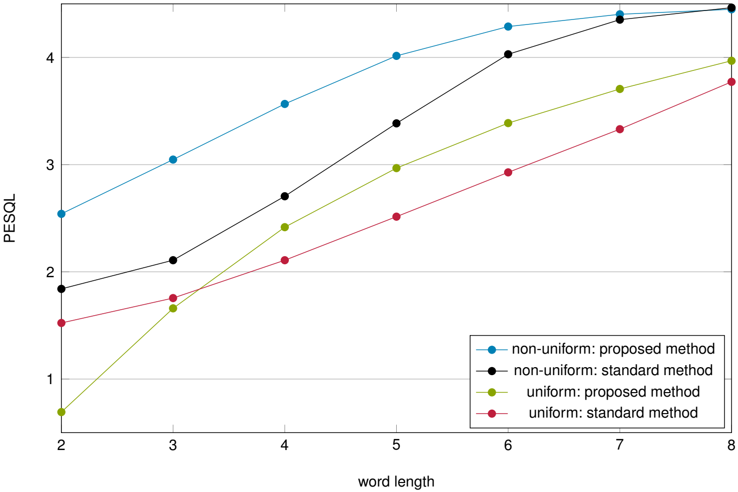
Numerical Experiments



PESQL

- *Perceptual Evaluation of Speech Quality*
- standard measure for speech quality experienced by the user of a telecommunication system
- sampled signal $\mathbf{f}$ is used as reference and compared to reconstructed signal $\mathbf{x}$
- compared PESQL of our reconstructions to the PESQL of standard reconstructions $Q_\Delta(\mathbf{f})$ and $C_A^{-1}(Q_A(\mathbf{f}))$
- results for $w = 2, \ldots, 8$ averaged over 720 speech signals from IEEE database

PESQL



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word length
non-uniform: proposed method
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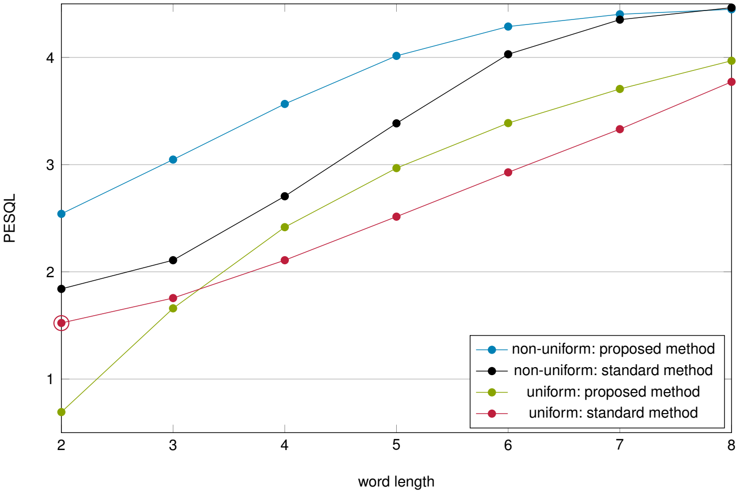
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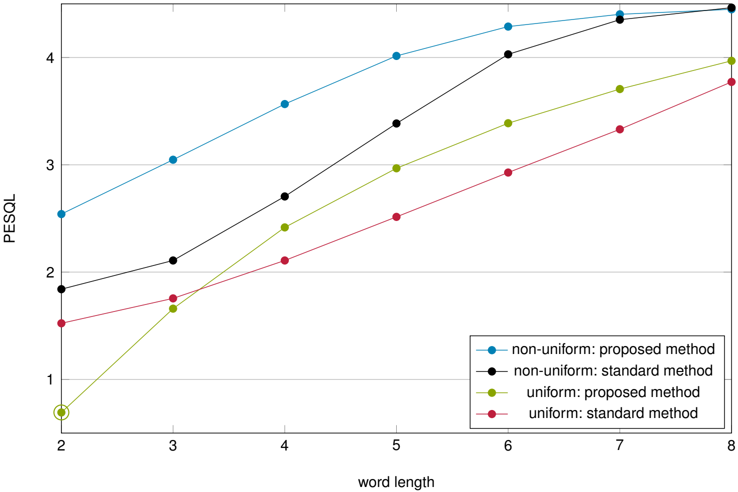
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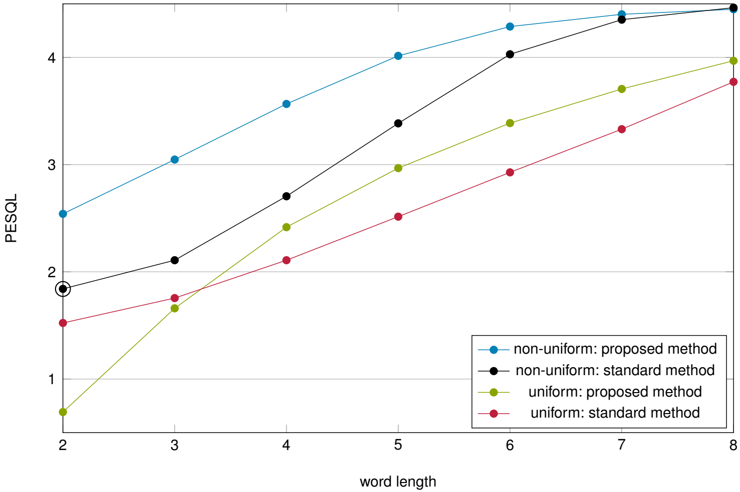
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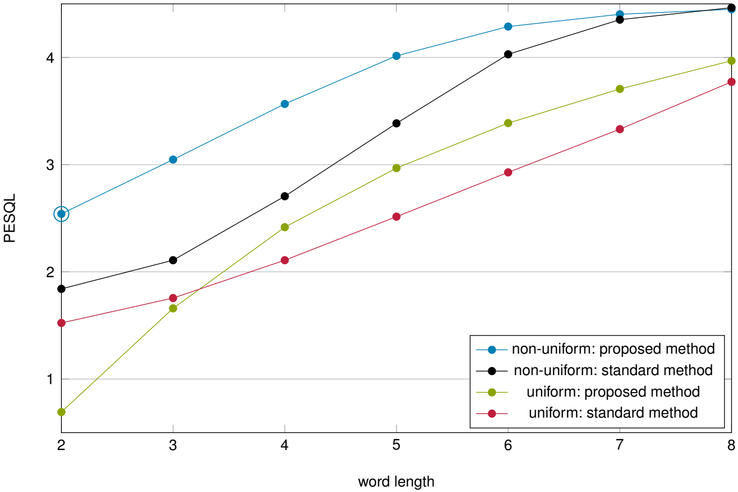
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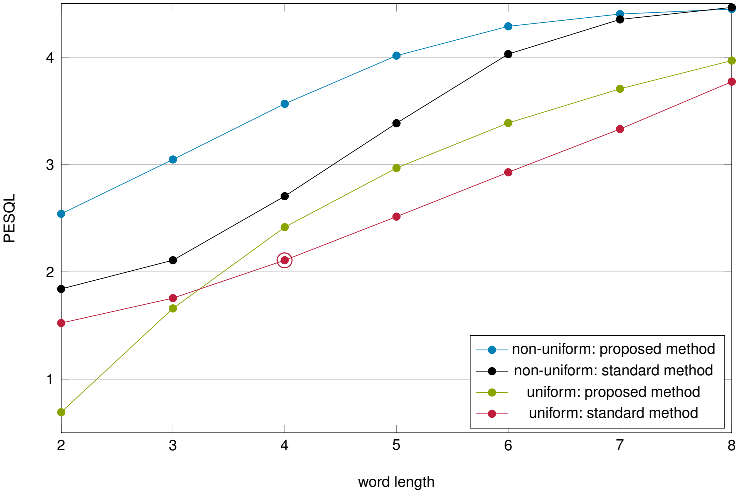
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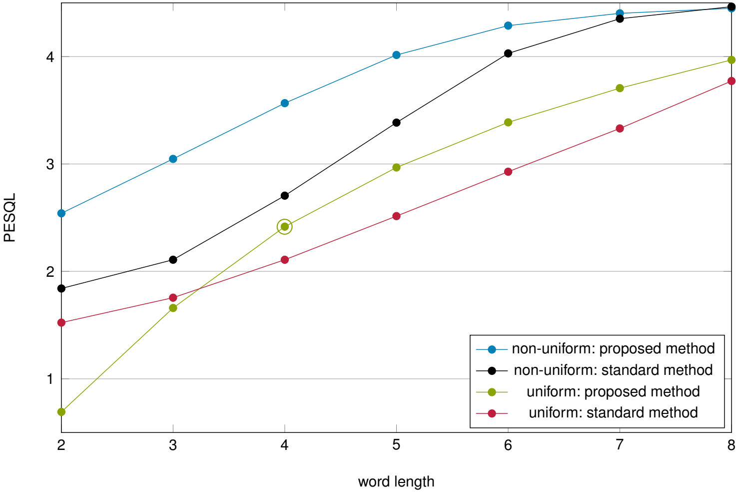
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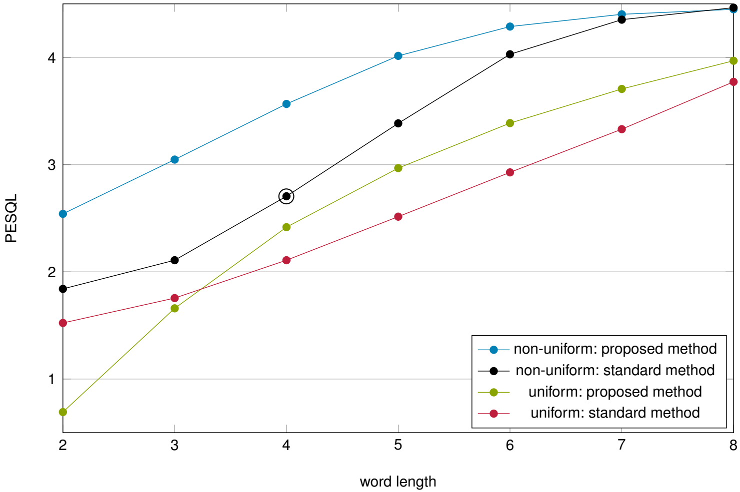

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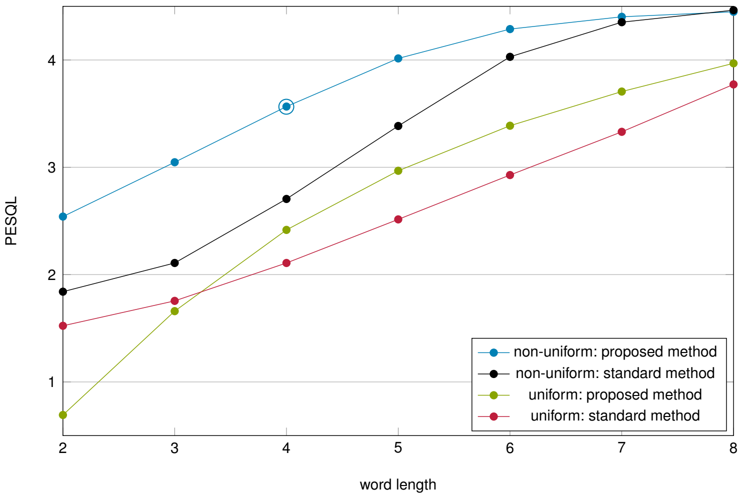
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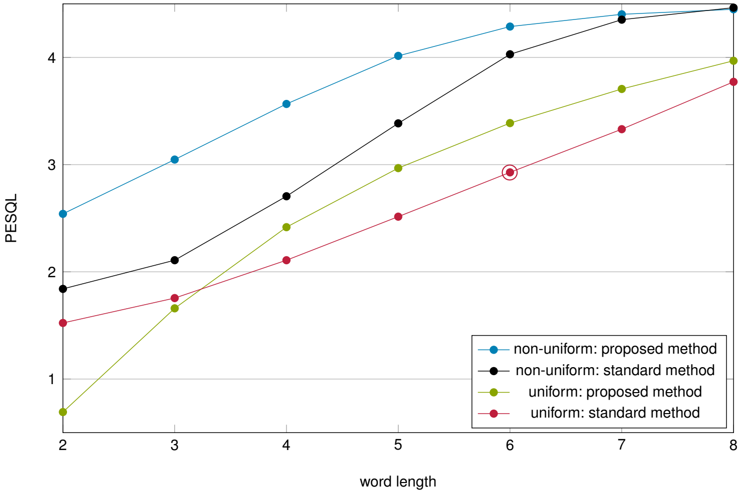

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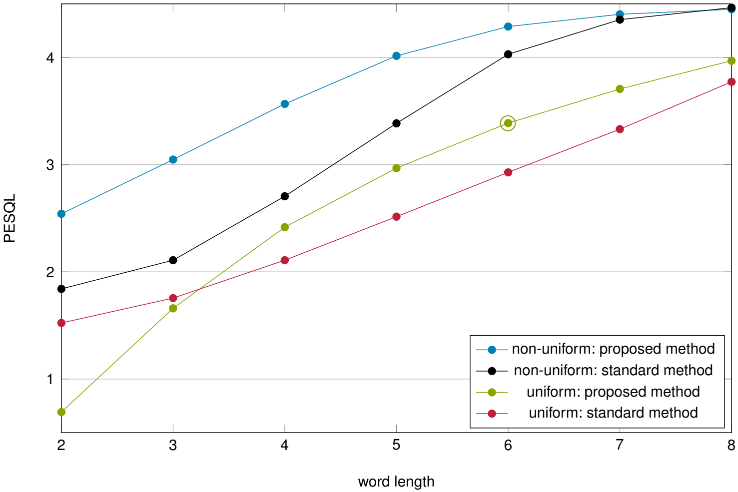

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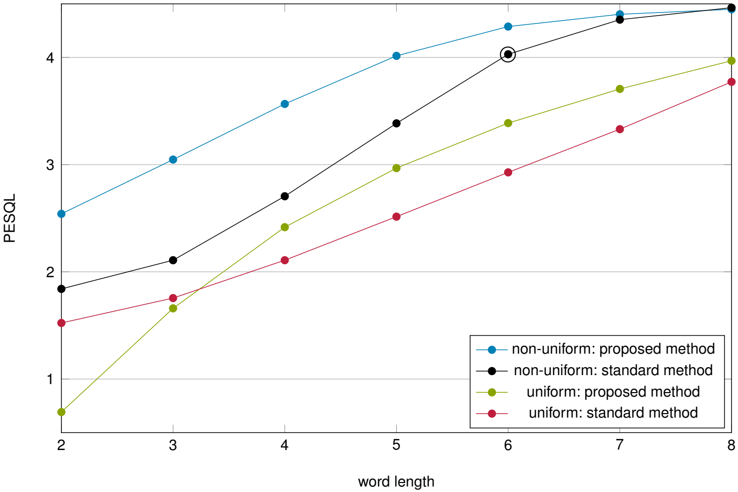
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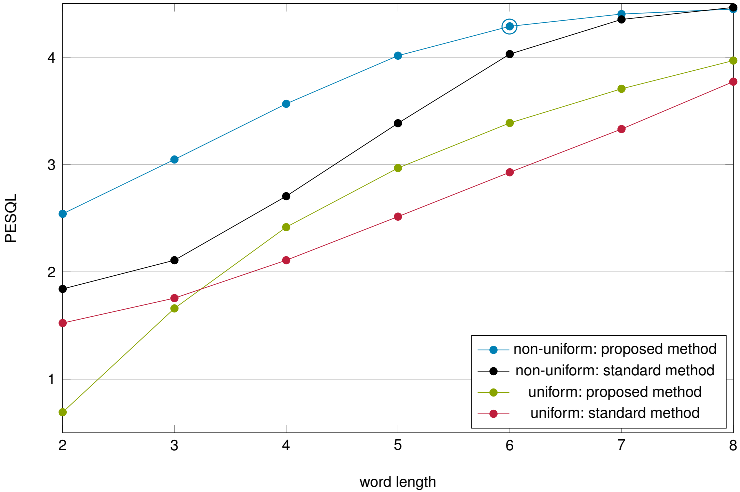
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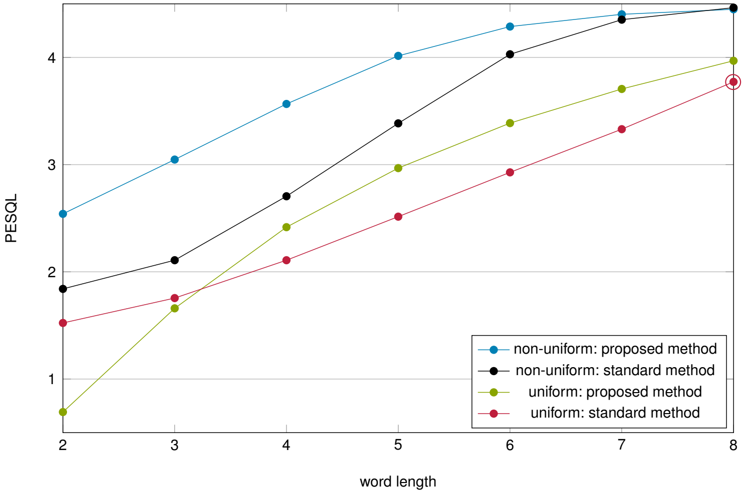
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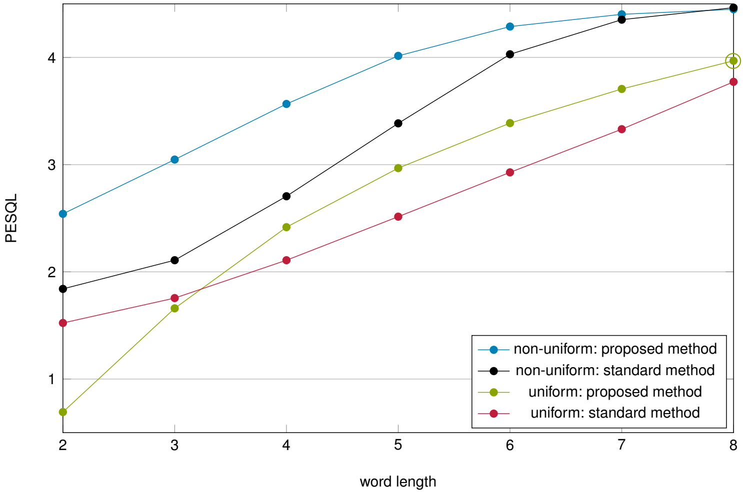

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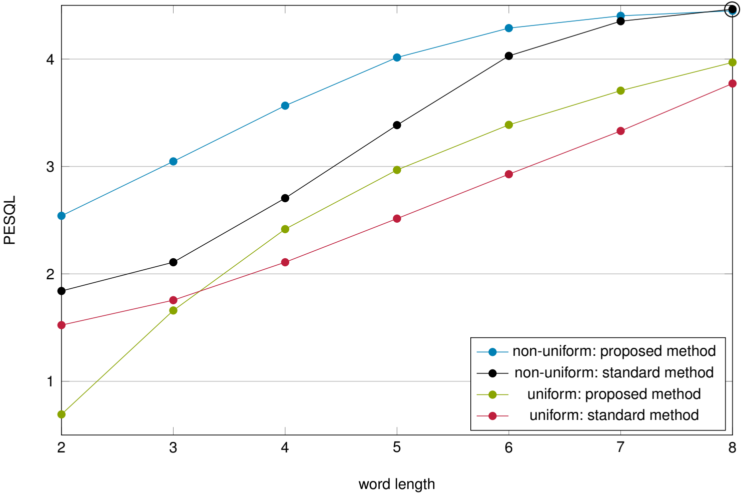
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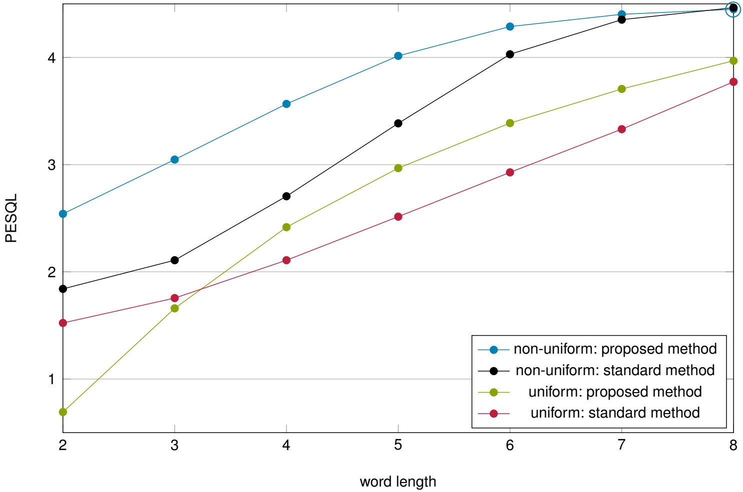
uniform: standard method

PESQL



PESQL
1
2
3
4
2
3
4
5
6
7
8
word length
non-uniform: proposed method
non-uniform: standard method
uniform: proposed method
uniform: standard method

Conclusion

- in *Wireless Acoustic Sensor Networks*, computational and power capacities are distributed asymmetrically between sensors and fusion center
- therefore, the proposed method features low encoding complexity and low datarate
- suitable sparsity priors and convex optimization can increase the percepted speech quality compared to standard methods
- real-time implementation is possible

Questions

Thank You for Your attention!

Slides available under https://www.tu-braunschweig.de/iaa/personal/brauer.