

Formelsammlung Ingenieurmathematik B
 Analysis 2

Binomische Formel

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

Richtungsableitung, $\mathbf{n} \in \mathbb{R}^d$, $\|\mathbf{n}\|_2 = 1$

$$\frac{\partial}{\partial \mathbf{n}} f(\mathbf{x}) = \lim_{h \rightarrow 0} \frac{1}{h} [f(\mathbf{x} + h\mathbf{n}) - f(\mathbf{x})]$$

$$\frac{\partial}{\partial \mathbf{n}} f(\mathbf{x}) = \nabla f(\mathbf{x}) \cdot \mathbf{n}$$

Satz von Schwarz

$$f_{,x_k x_\ell} \in C \quad \forall k, \ell \Rightarrow f_{,x_k x_\ell} = f_{,x_\ell x_k}$$

Totale Ableitung $A: \mathbb{R}^d \rightarrow \mathbb{R}^1$, $f: \mathbb{R}^d \rightarrow \mathbb{R}^1$

$$0 = \lim_{\mathbf{v} \rightarrow 0} \frac{f(\mathbf{x} + \mathbf{v}) - f(\mathbf{x}) - A\mathbf{v}}{\|\mathbf{v}\|_2}$$

Hesse-Matrix von $f: \mathbb{R}^d \rightarrow \mathbb{R}^1$

$$\nabla \nabla f(\mathbf{x}) = (f_{,x_k x_\ell})_{k, \ell=1}^d$$

Taylor-Reihe für $f: \mathbb{R}^1 \rightarrow \mathbb{R}^1$

$$T(x_0 + h) = f(x_0) + hf'(x_0) + \frac{1}{2!} h^2 f''(x_0) + \dots$$

Tangentialebene $T_1(\mathbf{x}) = T_1(\mathbf{x}_0 + \mathbf{v})$

Lagrange-Funktion von f zu Nebenbedingung $g_k(\mathbf{x}) = 0$, $k = 1, \dots, m$

$$L(\mathbf{x}, \lambda_1, \dots, \lambda_m) = f(\mathbf{x}) + \lambda_1 g_1(\mathbf{x}) + \dots + \lambda_m g_m(\mathbf{x})$$

Jacobi-Matrix von $\mathbf{g}: \mathbb{R}^d \rightarrow \mathbb{R}^m$:

$$\nabla \mathbf{g}(\mathbf{x}) \in \mathbb{R}^{m \times d}$$

Kettenregel $\mathbf{g}: \mathbb{R}^d \rightarrow \mathbb{R}^m$, $\mathbf{f}: \mathbb{R}^m \rightarrow \mathbb{R}^k$

$$\begin{aligned} \frac{\partial}{\partial \mathbf{x}} \mathbf{f}(\mathbf{g}(\mathbf{x})) &= \frac{\partial}{\partial \mathbf{g}} \mathbf{f}(\mathbf{g}(\mathbf{x})) \cdot \frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}} \\ &= \nabla \mathbf{f}(\mathbf{g}(\mathbf{x})) \cdot \nabla \mathbf{g}(\mathbf{x}) \end{aligned}$$

Divergenz von $\mathbf{v}: \mathbb{R}^d \rightarrow \mathbb{R}^d$

$$\operatorname{div} \mathbf{v} = \nabla \cdot \mathbf{v}$$

Rotation von $\mathbf{g}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$\operatorname{rot} \mathbf{g} = \nabla \times \mathbf{g}$$

Laplace-Operator

$$\Delta f = \nabla \cdot \nabla f$$

Parameterdarstellung von $\Gamma: \mathbb{R}^1 \rightarrow \mathbb{R}^d$

$$\Gamma = \{\gamma(t) \in \mathbb{R}^d, t \in [a, b] \subset \mathbb{R}^1\}$$

Tangentialvektor

$$\frac{d}{dt} \gamma(t) = \gamma'(t)$$

Schwerpunkt

$$\mathbf{x}_s = \frac{1}{m} \int_{\Omega} \mathbf{x} \rho(\mathbf{x}) \, dV, \quad m = \int_{\Omega} \rho(\mathbf{x}) \, dV$$

Trägheitsmoment

$$J = \int_{\Omega} r(\mathbf{x})^2 \rho(\mathbf{x}) \, dV$$

Steinerscher Satz $J_p = J_s + \|\mathbf{y}\|_2^2 \cdot m$

Kurvenintegral

$$\int_{\Gamma} f(\mathbf{x}) \, ds = \int_a^b f(\gamma(t)) \cdot \|\gamma'(t)\|_2 \, dt$$

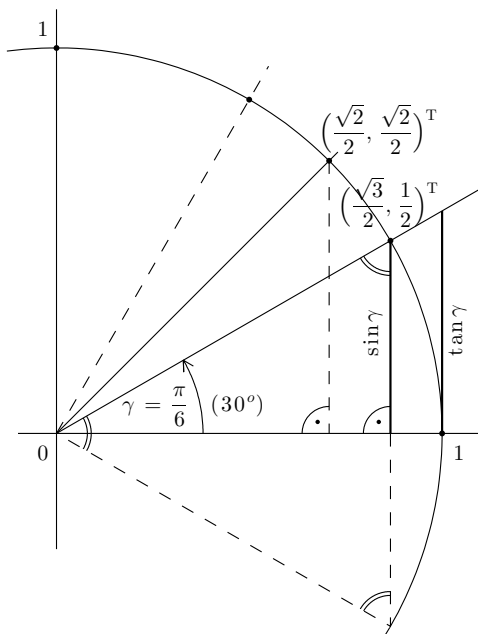
Arbeitsintegral

$$\int_{\Gamma} \mathbf{g}(\mathbf{x}) \cdot d\mathbf{x} = \int_a^b \mathbf{g}(\gamma(s)) \cdot \gamma'(s) \, ds$$

Eulersche Identität

$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

Trigonometrische Funktionen



Trigonometrische Beziehungen

$$1 = \sin^2 x + \cos^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\cos t = \frac{1}{2} (e^{it} + e^{-it})$$

$$\sin t = \frac{1}{2i} (e^{it} - e^{-it})$$

Fourier-Reihe

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos kx + b_k \sin kx = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$$

mit Fourier-Koeffizienten

$$a_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos kx \, dx$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin kx \, dx$$

$$c_k = \frac{1}{2\pi} \int_0^{2\pi} f(x) e^{-ikx} \, dx$$

Hyperbelfunktionen

$$\cosh t = \frac{1}{2} (e^t + e^{-t})$$

$$\sinh t = \frac{1}{2} (e^t - e^{-t})$$

$$\operatorname{arsinh} t = \ln(t + \sqrt{t^2 + 1})$$

$$\operatorname{arcosh} t = \ln(t + \sqrt{t^2 - 1})$$

Partialbruchzerlegung

$$\frac{p(x)}{(x-x_0)(x-x_1)} = \frac{A}{x-x_0} + \frac{B}{x-x_1}$$

$$\frac{p(x)}{(x-x_0)^k} = \frac{A_1}{x-x_0} + \frac{A_2}{(x-x_0)^2} + \dots + \frac{A_k}{(x-x_0)^k}$$

Differentiation

$$\text{Produktregel } (uv)' = u'v + uv'$$

$$\text{Kettenregel } \frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

$$(x^n)' = n \cdot x^{n-1}, \quad n \neq 0$$

$$(a^x)' = \ln a \cdot a^x, \quad a > 0$$

Partielle Integration

$$\int u(x)v'(x) \, dx = u(x)v(x) - \int u'(x)v(x) \, dx$$

Spezielle Integrale

$$\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}$$

$$\int_0^{\pi} \sin^2 x \, dx = \frac{\pi}{2}$$

$$\int \ln x \, dx = x \cdot \ln x - x + c, \quad c \in \mathbb{R}$$

$$\int x \sin kx \, dx = \frac{\sin kx - kx \cos kx}{k^2} + c$$

$$\int x \cos kx \, dx = \frac{kx \sin kx + \cos kx}{k^2} + c$$

$$\int x^2 \sin kx \, dx = \frac{(2 - k^2 x^2) \cos kx + 2kx \sin kx}{k^3} + c$$

$$\int x^2 \cos kx \, dx = \frac{(k^2 x^2 - 2) \sin kx + 2kx \cos kx}{k^3} + c$$

p-q-Formel für $x^2 + px + q = 0$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$