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Partitioned Methods for Multifield Problems:
Assignment 5:Aitken accelerator and partitioned Runge-Kutta method

Exercise 1: **(12 points)**

Given a series defined by

$$x(n) = \sum_{k=0}^{n} \frac{(-1)^k}{(1+2k)} \quad , \tag{1}$$

and knowing the series converges to $\lim_{n\to\infty} x(n) = \frac{\pi}{4}$. Write a Matlab program that speeds up the convergence by using Aitken Δ^2 method. Report the n values needed to reach specific accuracy by the $x(n)$ and by the Aitken series, taking error tolerance 10^{-2},10^{-3} and 10^{-4} respectively.

Exercise 2: **(24 points)**

Suppose we have two coupled initial value problems

$$\dot{\mathbf{u}} = \mathbf{f}(t, \mathbf{u}, \mathbf{v}) \tag{2}$$

$$\dot{\mathbf{v}} = \mathbf{g}(t, \mathbf{u}, \mathbf{v}) \tag{3}$$

with $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. We are going to solve the initial value problems with implicit Runge-Kutta (RK) method in which a Newton's method is adopted to solve the nonlinear system for each time step. We don't have the cross-derivatives $\mathbf{f}_v$ and $\mathbf{g}_u$ and would like to apply the Newton's method in a partitioned way, for this purpose we choose Gauss-Seidel scheme.

(a) Write out the expression of $\boldsymbol{K}_i$ in the RK procedure of problem (2), and also the expression of its counterpart (denoted as $\boldsymbol{I}_i$) in the RK procedure of problem (3). (4 points)

(b) Write out the system of equations for the partitioned Newton's method (which utilizes only $\mathbf{f}_u$ and $\mathbf{g}_v$), explain how the Gauss-Seidel iteration proceeds. (10 points)

(c) If n is so large that the memory is on a tight budget, which type of the implicit Runge-Kutta method can alleviate the problem? (4 points)

(d) What else methods can also proceed the implicit RK method in a partitioned way? (6 points)

Exercise 3: *(Optional, but important to know the answer)*

(a) Consider two coupled ODEs, and we run two order p ($p > 1$) solvers on each but couple them only weakly (say,block Jacobi or Gauss-Seidel way), what would be the order of accuracy of the solution? If we want the solution to have the same accuracy of order p, what kind of coupling should we use?

(b) In our implicit RK method (suppose it has an order p) in the above exercise, we know the $\mathbf{K}_i$ is a function of $\boldsymbol{u}$ and $\mathbf{v}$ (so is $\mathbf{I}_i$).

- If $\mathbf{K}_i$ at the n-th timestep is expressed in terms of $\boldsymbol{u}$ and $\mathbf{v}$ that are also at the n-th timestep, what is the order of the RK solution?
- On the contrary, if $\mathbf{K}_i$ at the n-th timestep is expressed in terms of $\boldsymbol{u}$ at the n-th timestep and $\mathbf{v}$ at the $(n-1)$-th time step, what is the order of the RK solution?
- What are the names of these two types of coupling?

(c) If we use a strong coupling between the two implicit RK solvers, and solve the nonlinear system for $\mathbf{K}$ and $\mathbf{I}$ at each time step with a partioned Newton's method using block Gauss-Seidel or block Jacobi iteration, does this change the order of the RK solution?