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Partitioned Methods for Multifield Problems:
Exercise 3: Partitioned Approach: Iterative Methods

Exercise 1: **(26 points)**

Consider two coupled initial value problems:

$$\begin{aligned} \dot{u} &= au - (u+v)/2 \\ \dot{v} &= av + (u+v)/2, \end{aligned}$$

write and run a Matlab script for a numerical solution of the coupled problems in a partitioned way. Use Heun's scheme for the time discretization. Take $a = 0.1$, $u(0) = v(0) = 1$, and compute the solution in time period $t \in [0, 5]$. Use weak coupling and strong coupling and compare their error convergence rates (to evaluate error of the numerical solutions, compare them with the analytical solution $u(t) = e^{at} - te^{at}$ and $v(t) = e^{at} + te^{at}$). The coupled equilibrium system arises in the strong coupling can be solved by using Jacobi or Gauss-Seidel iterations.

The error for the comparison can be taken as the error of the solutions at $t = 5$. Use different time steps τ to solve the problems, and observe how fast the errors are reduced by reducing τ, plot the "error vs. τ" curses for weak and strong coupling schemes respectively (better use logarithmic scale to see the difference).

Which coupling scheme has the 2nd-order error convergence rate?

Exercise 2: **(10 points)**

The block Jacobi method and the block Gauss-Seidel method shall be used to approximate a solution of the non-linear system

$$\begin{aligned} f(x,y) &= 0 \\ g(x,y) &= 0 \end{aligned}$$

with $x, y \in \mathbb{R}$ and $f, g : \mathbb{R}^2 \to \mathbb{R}$. The Matlab code `block-jacobi.m` implements the block Jacobi method, at which a fixpoint iteration with a damping is used to solve each subproblem of the non-linear system. An example for the non-linear system is provided by the Matlab code `f-ex.m`. The Matlab script `main.m` applies the implemented block Jacobi method onto the example.

Plot the relative error over the number of iterations. Do you see convergence? If not, adapt the damping parameters α and β, so that a convergence is achieved.
Implement Block-Newton method to solve the coupled system, check the Matlab code `dfdx-ex.m` to know the sub-matrices of the full Jacobi matrix. The method uses only the sub-matrices, does not inverse the full Jacobi.

Plot the "error vs. iteration" curves for both methods and compare their error convergence rates.