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Partitioned Methods for Multifield Problems:
Assignment 2: Strong coupling and block Gauss-Seidel iteration

Exercise 1: **(36 points)**

Consider the piston problem which involves interaction of a fluid dynamic system and a linear vibration system, after a discretization in space the problem can be stated in such a coupled system:

$$\begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{v}} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} + \begin{pmatrix} \mathbf{f}(t) \\ \mathbf{g}(t) \end{pmatrix}$$

with $\mathbf{u}$ and $\mathbf{v}$ the discretized unknowns in the two subsystems. The matrices $\mathbf{B}$ and $\mathbf{C}$ are the coupling terms. By using an implicit Euler scheme and a weak coupling, and omitting the f and g terms, the solutions at the n-th time step are

$$\mathbf{u}^{(n)} = \tau(\mathbf{A}\mathbf{u}^{(n)} + \mathbf{B}\mathbf{v}^{(n-1)}) + \mathbf{u}^{(n-1)},$$

$$\mathbf{v}^{(n)} = \tau(\mathbf{C}\mathbf{u}^{(n-1)} + \mathbf{D}\mathbf{v}^{(n)}) + \mathbf{v}^{(n-1)}.$$

τ is the time step size. This weakly coupled system is solved by a staggered method in the downloadable Matlab program.

Now consider using a strong coupling instead:

$$\mathbf{u}^{(n)} = \tau(\mathbf{A}\mathbf{u}^{(n)} + \mathbf{B}\mathbf{v}^{(n)}) + \mathbf{u}^{(n-1)},$$

$$\mathbf{v}^{(n)} = \tau(\mathbf{C}\mathbf{u}^{(n)} + \mathbf{D}\mathbf{v}^{(n)}) + \mathbf{v}^{(n-1)}.$$

Suppose we don't have the monolithical solver for the system as a whole, we can solve for the $\mathbf{u}^{(n)}$ and $\mathbf{v}^{(n)}$ by a Gauss-Seidel iteration at every time step, taking $\mathbf{u}^{(n-1)}$ and $\mathbf{v}^{(n-1)}$ as the initial guess.

Write a Matlab program that solves the above strongly coupled system by a Gauss-Seidel iteration at every time step, using the same data and settings given in the code for the staggered method. Adjust the τ to get a convergence.