

Introduction to Scientific Computing
Solving difference equations
Assignment 3

Exercise 1: (20 points)

Consider the following difference equations:

- 1) $x_{n+2} - 9x_n = 2^n$, with initial condition: $x_0 = 0, x_1 = 1$.
- 2) $x_{n+2} + 4x_{n+1} + 4x_n = 2^n$, with initial condition: $x_0 = 1, x_1 = 2$.

- (a) For each of the difference equations, find the complete solutions of their homogeneous versions, without matching initial conditions. (6 points)
- (b) Find the particular solutions of the inhomogeneous equations respectively. (Instruction: use the ansatz from Table 2.5 in the ODE script which is at the end of section 2.2). (6 points)
- (c) Write down their general solution respectively. (2 points)
- (d) Match the initial conditions. (6 points)

Exercise 2: (16 points)

Given a linear system of difference equation:

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n$$

- (a) Write out the algebraic solution. (4 points)
- (b) Let

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

and $\mathbf{x}_0 = (1, 1)^\top$, write a Matlab program to compute the algebraic solution at $n = 10$ by using eigen decomposition. (12 points)