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Introduction to Scientific Computing

Iterative method
Assignment 5

Exercise 1: *(Stability of Eulor method)* **(12 points)**

Suppose we use Euler's finite difference scheme to solve Heat equation. I.e. for $0 < x < 1,\ t > 0$ we have

$$u_t = u_{xx} \tag{1}$$

with initial and boundary conditions:

$$u(x, t = 0) = u_0(x), \quad u(0, t) = u(1, t) = 0.$$

Let the spacial and temporal domains be discretized by mesh x_m and t_n:

$$x_m = mh, \quad m = 0, 1, \cdots, M, \quad h = 1/M;$$
$$t_n = n\kappa, \quad n = 0, 1, \cdots, N, \quad \kappa = T_{max}/N;$$

where T_{max} is the maximum time length, and let U_m^n be the solution computed at node (x_m, t_n). We approximate u_t by $(U_m^{n+1} - U_m^n)/\kappa$, and approximate u_{xx} by $(U_{m+1}^n - 2U_m^n + U_{m-1}^n)/h^2$.

(a) Let $r = \kappa/h^2$, write out the matrix $\mathbf{A}$ in the Euler scheme that solves the Heat equation (1):

$$\mathbf{U}^{n+1} = \mathbf{A}\mathbf{U}^n \tag{2}$$

where $\mathbf{U}^n = (U_1^n, U_2^n, \cdots, U_{M-1}^n)^\top$. (4 points)

(b) Analyse the stability of this scheme, specify under which condition the scheme converges.
Hint: Use the formula for the eigenvalues of tridiagonal Toeplitz matrices. (8 points)

Exercise 2: *Jacobi and Gauss-Seidel methods* **(24 points)**

Download the Matlab code in "Truss_Analysis.zip". Initialize data by running the command "D = Data( )" , and run "ST.m" to solve for the displacement of the truss nodes. Run "TP.m" to watch the visualization of the deformed truss(with the scaler "Sc" set to 50-100).(the last step is not obligatory)

(a) Replace the solution step in ST.m (in line 46) by Jacobi and Gauss-Seidel methods. (8 points)

(b) Run each of the two iterative methods for 50 iterations. On each step compute their 2-norm errors by comparing to the original solution. Draw the curves of errors. (8 points)

(c) Analyse the stability of the two methods by checking their spectral radius, and explain the results observed in the above subtask. (8 points)