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Introduction to Scientific Computing

Fixed Points and Stability
Assignment 4

Exercise 1: **(30 points)**

Let be given a second order difference equation:

$$x_{n+2} - \frac{5}{6}x_{n+1} + \frac{1}{6}x_n = 2$$

(a) Find the general solution (as an appropriate combination of the solutions of the corresponding homogeneous equation and the particular solution of the inhomogeneous one). (4 points)

(b) Find the fixed point x_* of the equation. What is the relationship between x_* and the particular solution found in the last step? (2 points)

(c) Write a Matlab program to compute the sequence from x_3 to x_{50}, setting the initial values x_0 and x_1 to be arbitrary real numbers. Visualize the sequence. (8 points)

(d) What do you observe about the behaviour of the sequence? Explain the reason for that. (4 points)

(e) Write the second order difference equation into the form of a first order linear system of equations, i.e. $\mathbf{y}_{n+1} = \mathbf{A}\mathbf{y}_n + b$. (4 points)

(f) Explain why the eigenvalues of $\mathbf{A}$ and the zeros of the characteristic polynomial of the second order equation are the same. (4 points)

(g) Analyse the stability of the fixed point by checking the eigenvalues of the Jacobian $\mathbf{A}$. (4 points)

Exercise 2: **(6 points)**

A fixed-point iteration $x_{n+1} = \phi(x_n)$ is defined by

$$\phi(x) = e^{-x}.$$

Verify that $\phi(x)$ meets the conditions of Banach's fixed point theorem for the interval [0.5,0.69].