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Introduction to Scientific Computing
ASSIGNMENT 2

Exercise 1: *Machine Accuracy* **(10 points)**

(a) Write a Matlab program to identify the machine accuracy of your computer, that is: find the smallest number ϵ_a that can be effectively plus to a. Try with $a = 1, 2, 3, \cdots, 32$ (at least).
Instructions: write a while-loop, in which ϵ_a keeps on getting divided by 2 as long as $a+\epsilon_a-a > 0$ holds. The start-value for ϵ_a in this procedure should be chosen to be 1. (6 points)

(b) Explain why ϵ_a is proportional to the power of 2 (i.e. 2^n) that is smaller than and nearest to a. (4 points)

Exercise 2: **(10 points)**

(a) Write out the expression of the condition number of the function

$$f(x, y) = x + y$$

with respect to error in x, and briefly discuss under which situation the condition number would be relativey larger. (4 points)

(b) Write out the expression of the condition number of the function

$$f(x) = 2x^2 + x - 1,$$

and discuss that around which values of x the function is ill-conditioned, and explain the reason. (6 points)

Exercise 3: **(16 points)**

(a) Prove the 2-norm of a symmetric matrix is the absolute value of its largest eigenvalue.
Instructions: Use the definition of matrix's 2-norm, i.e.

$$\|\mathbf{A}\|_2 = \sup_{\|\mathbf{x}\|_2=1} \|\mathbf{A}\mathbf{x}\|_2,$$

and write $\mathbf{x}$ as a linear combination of $\mathbf{A}$'s eigenvectors, also utilize the orthogonality of the eigenvectors. (4 points)

(b) Compute the condition number of the below matrix by hand.

$$\mathbf{A} = \begin{pmatrix} 100 & 0 \\ 0 & 1 \end{pmatrix}$$

(6 points)

(c) Given a linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

in which $\mathbf{A}$ is defined as above,

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

perturb $\mathbf{b}$ by

$$\Delta\mathbf{b} = \begin{pmatrix} 0 \\ 0.01 \end{pmatrix},$$

and compute the relative errors between the non-perturbed and perturbed solutions (in term of 2-norm), what do you observe? (6 points)