

Introduction to Scientific Computing ASSIGNMENT 1

Exercise 1: Gauss elimination (1 points)

Solve the below system of linear equations by a Gauss elimination, writing out the process step by step.

$$x + 2y = 8$$

$$2x - y = 1$$

Exercise 2: vector norm (1 points)

Compute the 1-norm, 2-norm and ∞ -norm of the vector $\mathbf{x} = (1, 2, 2)$, i.e. $\|\mathbf{x}\|_1$, $\|\mathbf{x}\|_2$ and $\|\mathbf{x}\|_\infty$.

Exercise 3: matrix norm (1 points)

Compute the induced norm $\|\mathbf{A}\|_1$ and $\|\mathbf{A}\|_\infty$ of the below matrix \mathbf{A} .

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

Exercise 4: eigenvalues (2 points)

(a) Compute all the eigenvalues of the forementioned matrix \mathbf{A} (1 points)

(b) We know $\mathbf{v}_1 = (1, -1, 1)$ is one of the eigenvectors of the below matrix \mathbf{B} , compute the eigenvalue that is associated to \mathbf{v}_1 .

$$\mathbf{B} = \begin{bmatrix} 11 & 1 & -1 \\ 1 & 6 & -4 \\ -1 & -4 & 6 \end{bmatrix}$$

(1 points)

Exercise 5: differentiation (1 points)

Write out the first order derivative of $f_1(x) = x^3 + x^2 + 1$ and $f_2(x) = e^{2x}$.

Exercise 6: Integration (1 points)

Write out the integrations $\int x^2 dx$ and $\int e^{2x} dx$

Exercise 7: Interpolation (2 points)

Identify the polynomial $y = ax^2 + bx + c$ that interpolates the (x, y) points $(1, 2)$, $(2, 7)$ and $(3, 14)$.

Hint: Using the fact that when x takes the values 1, 2 and 3, y takes the values 2, 7 and 14, to form three linear equations of a , b and c and solve it by Gauss elimination.