Institute of Scientific Computing
Technical University Braunschweig
Noemi Friedman, Ph.D.

Winter Term 2017
Due date: 8. Nov. 2016

Introduction to PDEs and Numerical Methods (PDEs 1): Assignment 1 (50 points)

Exercise 1: *Differential operators* **(15 points)**

(a) Let $f_1(x, y, z) = x^2 e^{(-3y)} \cos(2z)$. Determine $\frac{\partial f_1}{\partial x}$, $\frac{\partial f_1}{\partial y}$, $\frac{\partial f_1}{\partial z}$ and ∇f_1. (4 points)

(b) Let $\mathbf{f_2}(x, y, z) = (\cos(xy), xy, e^{(2z)})^T$. Determine $\nabla \cdot \mathbf{f_2}$ and $\nabla \times \mathbf{f_2}$. (4 points)

(c) Determine Δf_1 (see the function f_1 in subtask (a)). (3 points)

(d) Show that $\nabla \cdot \nabla f = \Delta f$ and $\nabla \times \nabla f = 0$ for any two-times differentiable function $f : \Omega \to \mathbb{R}^3$. (4 points)

Exercise 2: *Heat equation* **(17 points)**

Consider the heat equation on a bar of unit length, with parameter $\beta^2 = \frac{\lambda}{\rho c}$:

$$\frac{\partial}{\partial t}\theta(x, t) - \beta^2 \frac{\partial^2}{\partial x^2}\theta(x, t) = f(x, t)$$

(a) Assume boundary conditions $\theta(0, t) = 0$, $\theta(\pi, t) = 0$ and the source term $f(x, t) = \sin(x)$. Prove that $\theta(x, t) = \sin(x)$ can be a solution of the heat equation and specify the value of β that ensures this proof. (5 points)

(b) Now assume $\beta^2 = 4$, boundary conditions $\theta(0, t) = \theta(1, t) = 0$ and a solution $\theta(x, t) = (t^2 + \frac{1}{2}) \sin(\pi x)$. What must $f(x, t)$ look like if the heat equation should be satisfied. (7 points)

(c) Prove that $\theta(x, t) = t + \frac{1}{2}x^2$ is a solution of the heat equation. Write down the corresponding boundary and initial conditions. (5 points)

Exercise 3: *Classification of differential equations* **(8 points)**

Classify (order, linear/nonlinear, stationary/instationary, homogeneous, inhomogeneous) the following differential equations:

(a)

$$\frac{\partial^3 u}{\partial x^3} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = 0$$

(4 points)

(b)

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \sin(u) = x \sin(t)$$

(4 points)

Exercise 4: *Classification of differential equations 2* **(10 points)**

(a) Determine and sketch the subsets of $\mathbb{R}^2$, where the following equations are elliptic/-parabolic/hyperbolic:

$$u_{xx} + 2u_x + (1 - y^2)u_{yy} + u = 0$$

(5 points)

(b) Determine whether the following equations are elliptic, parabolic or hyperbolic:

$$u_{xx} - u_{xy} + 2\, u_y + u_{yy} - 3\, u_{yx} + 4u = 0$$

$$9\, u_{xx} + 6\, u_{xy} + u_{yy} + u_x = 0$$

(5 points)