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Advanced Methods for ODEs and DAEs: *Assignment 2*

Exercise 1: **(36 points)**

Consider two masses $m_1 = 2$kg and $m_2 = 2$kg on the distance $r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ attracted by gravitational force gm_1m_2/r^2, where $g = 2m^3kg^{-1}s^{-2}$ is the gravitational constant (artificial value!). The mass movement can be described by a second Newton law via four coupled equations

$$m_1 \frac{dx_1^2}{dt^2} = \frac{gm_1m_2}{r^3}(x_2 - x_1)$$
$$m_1 \frac{dy_1^2}{dt^2} = \frac{gm_1m_2}{r^3}(y_2 - y_1)$$
$$m_2 \frac{dx_2^2}{dt^2} = \frac{gm_1m_2}{r^3}(x_1 - x_2)$$
$$m_2 \frac{dy_2^2}{dt^2} = \frac{gm_1m_2}{r^3}(y_1 - y_2)$$

Integrate the previous system of equations in time $[0, 25]$ by using general Kronecker tensor product approach Runge Kutta method and Gauss-Legendre Runge Kutta coefficients:

$$\begin{array}{c|cc} \frac{1}{2} - \frac{1}{6}\sqrt{3} & \frac{1}{4} & \frac{1}{4} - \frac{1}{6}\sqrt{3} \\ \frac{1}{2} + \frac{1}{6}\sqrt{3} & \frac{1}{4} + \frac{1}{6}\sqrt{3} & \frac{1}{4} \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

The initial conditions are given as:

$$x_1 = -1, \dot{x}_1 = 0, y_1 = 0, \dot{y}_1 = -1, x_2 = 1, \dot{x}_2 = 0, y_2 = 0, \dot{y}_2 = 1$$

The time step size should be chosen appropriate to this problem. The nonlinear system of equations should be solved by a Newton method, and the linear system can be solved by using built in Matlab function pcg or backslash.