

Advanced Methods for ODEs and DAEs:
Assignment 2

Exercise 1:

(36 points)

Consider solving an ODE of a Dahlquist problem

$$\frac{dx}{dt} = \beta x \tag{1}$$

with $\beta = -1$, $x(0) = 1$.

Write a Python class to solve the ODE (1) by a 2-stage Gauss-Legendre Runge Kutta method. The Butcher's array is given below.

$$\begin{array}{c|cc} \frac{1}{2} - \frac{1}{6}\sqrt{3} & \frac{1}{4} & \frac{1}{4} - \frac{1}{6}\sqrt{3} \\ \frac{1}{2} + \frac{1}{6}\sqrt{3} & \frac{1}{4} + \frac{1}{6}\sqrt{3} & \frac{1}{4} \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

You can base your work on the Python class in the last exercise, and build a new class for the Gauss-Legendre RK method by adding the needed implicit features.

Run the program to get the solution for a sufficient length of time period, and plot the $x(t)$ curve.