

Advanced Methods for ODEs and DAEs:
Assignment 4

Exercise 1:

(36 points)

Apply the two-stage Gauss method to the following ODE

$$\begin{aligned}\dot{y} &= \lambda y + \exp^{-t}, \quad \lambda \in (-\infty, -2], \\ y(0) &= -\frac{1}{1+\lambda}\end{aligned}$$

(a) Given exact solution

$$y(t) = -\frac{1}{1+\lambda} \exp^{-t},$$

evaluate the local absolute error of the first step of Gauss method. Make the table of the local error with respect to the step size $h = 2^{-n}$, $n = 0, 1, \dots, 8$ and the value of $\lambda = \{-2, -10, -100, -10^5, -10^9\}$.

(14 points)

(b) For each pair of h and λ estimate the order of the Gauss scheme numerically and present obtained results in the table.

(14 points)

(c) Elaborate previously obtained results.

(8 points)