

Advanced Methods for ODEs and DAEs:
Assignment 1

Exercise 1: **(12 points)**

Derive the second order explicit Runge Kutta method given by Butcher table

$$\begin{array}{c|cc}
 0 & 0 & 0 \\
 2/3 & 2/3 & 0 \\
 \hline
 & 1/4 & 3/4
 \end{array}$$

Exercise 2: **(24 points)**

(a) In Matlab implement general explicit Runge Kutta method given matrix A , and vectors b and c . Note that in case of explicit method the matrix A is lower triangular without diagonal. (8 points)

(b) For the system of ODEs

$$\begin{aligned}
 \dot{x} &= x + 2y \\
 \dot{y} &= 3x + 2y
 \end{aligned}$$

use the previously developed method to integrate the state $\mathbf{x} = (x, y)^T$ in time interval $[0, 100]$ given the Butcher table

$$\begin{array}{c|cccc}
 0 & 0 & 0 & 0 & 0 \\
 1/2 & 1/2 & 0 & 0 & 0 \\
 1/2 & 0 & 1/2 & 0 & 0 \\
 1 & 0 & 0 & 1 & 0 \\
 \hline
 & 1/6 & 1/3 & 1/3 & 1/6
 \end{array}
 ,$$

initial condition $\mathbf{x} = (1, 1)^T$ and the time step size $h = 1$. (8 points)

(c) Could you estimate the order of the previously given method without computing the local error? (8 points)