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Assignment 6
Due date: 30.11.2018

Introduction to Scientific Computing

Exercise 1: *Fixed point iteration* **14 points**

Given the function $f(x) = x^3 - x - 1$ find the roots of the function f on the $[1,2]$ using fixed point iteration. Show that the function $g(x_*) = x_*$ (which you have chosen to solve the problem) satisfies requirements of the Banach fixed point theorem. Illustrate the convergence process with a sketch. Implement the corresponding Matlab algorithm.

Exercise 2: *Lipschitz continuity* **12 points**

Given the following ODEs check wether the function g is a Lipschitz continuous function on the corresponding interval:

(a) $\dot{x} = g(x) = kx + \sqrt{x^2 + c}$, $k > 0$, $c > 0$, on the interval $[-5,5]$, (6 points)

(b) $\dot{x} = g(x) = \sqrt{x}$ on the interval $[0,5]$. (6 points)

Exercise 3: *Newton method* **10 points**

Solve the following nonlinear system

$$g_1(x_1,x_2) = \frac{1}{10}[1 - 10x_1 - x_2 - \sin(x_1 + x_2)] = 0$$

$$g_2(x_1,x_2) = \frac{1}{10}[2 + x_1 - 10x_2 + \cos(x_1 - x_2)] = 0$$

in $\mathbb{R}^2$ by using Newton method, starting from the initial point (0, 0) and satisfying the convergence criteria

$$\frac{||x_k - x_{k-1}||}{||x_{k-1}||} \leq 10e-6$$

Implement Newton function as implied by the unittests in the svn. As linear solver, use Gauss-Seidel or Jacobian, each method has to be written in one function, you can test versus Matlab backslash operator.