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Winter Term 2018/19
Assignment 3
Due date: 09.11.2018

Introduction to Scientific Computing

Exercise 1: *Equilibrium* **13 points**

Find equilibrium point for the following systems or equations and answer the question.

(a) The Lotka-Volterra equations (5 points)

$$\frac{dx}{dt} = \alpha x - \beta xy,$$

$$\frac{dy}{dt} = \delta xy - \gamma y.$$

Explain what kind of process is represented by the Lotka-Volterra equations? What is described by $\frac{dx}{dt}$ and $\frac{dy}{dt}$?

(b) The ordinary differential equation: $\dot{x} = \lambda x$. For what λ is the equilibrium point stable according to the Lyapunov definition? (3 points)

(c) The logistic equation: $\dot{x} = \lambda x(x - x_{max})$. For what a is the equilibrium point (3 points) stable according to the Lyapunov definition? Sketch the vector field of the equation.

(d) The difference equation: $x_{n+1} = ax_n$. For what a is the equilibrium point (2 points) stable according to the Lyapunov definition?

Exercise 2: *Transformations. Use parameter names as in brackets.* **18 points**

(a) Transform the following difference equation from 2nd order (3 points) to 1st order form:

$$x_{n+1} = 3x_n - 2x_{n-1} + 1.$$

(b) Having a two-mass (m_1, m_2) two-spring (c_1, c_2) system, sketch the system, explain the process and write down the differential equations describing the system. Transform the second order differential equations into a system of equations of first order. (10 points) (You might find it easier to choose the coordenate systems moving with the masses.)

(c) Having a damped (d_1, d_2) two-mass (m_1, m_2) two-spring (c_1, c_2) system, sketch the system, explain the process and write down the differential equations describing the system. Transform the second order differential equations into a system of equation of first order. (5 points)

Exercise 3: *Extreme values* **5 points**

Find extreme values of the function $u : \mathbb{R}^3 \to \mathbb{R}$ with

$$u(x_1, x_2, x_3) = \sin x_1 + \sin x_2 + \sin x_3 - \sin(x_1 + x_2 + x_3).$$

Are they minima, maxima or saddle points?