Institute of Scientic Computing
Technische Universität Braunschweig
Dr. Thilo Moshagen
Dr. H. Veselovska

Winter Term 2018/19
Assignment 2
Due date: 02.11.2018

Introduction to Scientific Computing

Remark: You are supposed to study the slides about condition number from the web site in the last week of Oktober. Some exercises refer to that.
You have to hand in this assignment, even empty, because it defines the groups.
Furthermore, you have to register at studIp! Use your Y-Number andaccording password. Do not forget this.

Exercise 1: *Analysis* **10 points**

(a) Let $f_i : \mathbb{R}^2 \to \mathbb{R}$, $i = 1,2,3$, with $f_1(x_1,x_2) = x_1^2 + x_2^2$, $f_2(x_1,x_2) = x_1^2 - x_2^2$, $f_3(x_1,x_2) = \frac{1}{2}x_1^2 + x_2^2$. Draw the contour lines and graphs of the functions f_i for all $i = 1,2,3$. (4 points)

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ with $f(x_1,x_2) = x_1^2 + x_2^2 + x_1x_2$.

Compute the Taylor-polynomial $T_2(\mathbf{x}_0 + \mathbf{x})$ of the function f at the $\mathbf{x}_0 = (0,2)^{\mathrm{T}}$. What is the difference between the function f and the Taylor-polynomial T_2 (6 points)

Exercise 2: *Bank account example* **5 points**

The monthly payment to the bank is 130 Euro and the bank interest is $q = 4.5\%$ per year. Each month the sum on the bank account is recomputed.

(a) Write the difference equation, which describe the dynamical model above. (2 points)
(b) Find the stationary points. Are they stable or unstable? Why? (3 points)

Exercise 3: *Floating-point arithmetic and condition number* **12 points**

Answer the following questions:

(a) What are a mantissa, an exponent and digits for some real number $x \in \mathbb{R}$? (4 points)
(b) What is a difference between the single-precision and double-precision floating-point numbers? (4 points)
(c) What kind of problems occur when substracting similar numbers? Consider for example

$$\begin{array}{r} 1.234567 \\ -1.234556 \\ = \end{array}$$

Assume that the last digit, 6, is wrong. (4 points)

Exercise 3: *The properties of eigenvectors and eigenvalues* **9 points**

Answer the following questions:

(a) What kind of matrix is called a Hermitian matrix, a skew-Hermitian? (3 points)

(b) What kind of properties of eigenvectors and eigenvalues of a Hermitian matrix have? (3 points)

(c) What kind of properties of eigenvectors and eigenvalues of a skew-Hermitian matrix have? (3 points)