Institute of Scientic Computing
Technische Universität Braunschweig
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Winter Term 2018/19
Assignment 1
Due date: 26.10.2018

Introduction to Scientific Computing
Linear Algebra

Exercise 1: *Vector spaces* **10 points**

(a) Prove that the set Π_n of all real polynomials of degree not exceeding n, form a vector space. How do the zero element and identity element look like? (3 points)

(b) Prove that the set of real polynomials of degree n does not form a vector space. (3 points)

(c) Let C be a set of all bounded functions $f\colon [a,b] \to \mathbb{R}$. How can one define the addition and multiplication on C to make C a vector space? Prove that according to your definition the set C is a vector space. (4 points)

Exercise 2: *Norms, eigenvalues and eigenvectors* **10 points**

(a) Given the vector $\mathbf{x} = (1,2,2)$ find both $\mathbf{L}^p$-norm, for $p = 1,2$, and $\|\mathbf{x}\|_\infty$. (2 points)

(b) Given the matrix $\mathrm{A} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ find $\|A\|_1$, $\|A\|_\infty$, eigenvalues and eigenvectors of A. (3 points)

(c) Given the matrix $\mathrm{B} = \begin{bmatrix} 1 & 2 & -3 \\ -5 & 1 & -4 \\ 0 & -2 & 4 \end{bmatrix}$ find eigenvalues and eigenvectors of B. (5 points)

Exercise 3: *Eigen decomposition* **16 points**

Let the linear map $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ with $\mathbf{x} \mapsto \mathbf{y} = \mathrm{C}\mathbf{x}$ be defined by the matrix $\mathrm{C} = \begin{bmatrix} 3 & -2 & 2 \\ 2 & -1 & 2 \\ 2 & -2 & 3 \end{bmatrix}$, where $\mathbf{x} = (x_1, x_2, x_3), \mathbf{y} = (y_1, y_2, y_3) \in \mathbb{R}^3$.

(a) Find the eigen decomposition of the matrix C. (4 points)

(b) Compute the determinant of the matrix C using its eigenvalues, and find whether the linear map φ provides one to one correspondence or not. (2 points)

(c) Compute eigenvalues of the matrix C^{-1} describing the inverse map φ^{-1} . (2 points)

(d) Compute the matrix C^5 that determines the map φ^5. (4 points)

(e) Compute the matrix C^{3015} that determines the map φ^{3015}. (4 points)