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Introduction to Scientific Computing:
IS A PERIODIC SOLUTION STABLE?

Due date: Fr. 18.1. 2019.

Exercise 1: *The limit to resonance* **(36 points)**

Consider the below IVP .

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}-y + x(1-x^2-y^2)\\ x + y(1-x^2-y^2)\end{pmatrix} \qquad x(0) = \begin{pmatrix}1\\0\end{pmatrix}$$

(a) This problem should remind you of a model problem we used when discussing stable and unstable solutions of IVP. Which and why? (2 points)

(b) Find the periodic solution $\mathbf{x}^*(t)$. You need Pythagoras at the unit circle. (10 points)

(c) Is the Poincaré map for $\mathbf{x}^*$ stable?
To find the monodromy matrix, prove that one solution of monodromy's matrix ODE is given by

$$\mathbf{v}_2(t) = \begin{pmatrix}\cos t\\ \sin t\end{pmatrix} e^{-2t}.$$

and remember from lecture that $\dot{\mathbf{x}}$ is an eigenvector of the Poincare map. (18 points)

(d) You learned that the center of the sun is a stable equilibrium point of the solar system. Does this necessarily mean that all Kepler IVP have stable periodic soluions? (6 points)