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Introduction to Scientific Computing: ENFORCED OSCILLATIONS

Due date: Fr. 11.1. 2019.

Exercise 1: *The limit to resonance* **(20 points)**

Consider the below spring-mass problem with an excitation term $\sin(\omega\, t)$.

$$\frac{d^2x}{dt^2} + kx = \sin(\omega, t), \qquad x(0) = 0, \left.\frac{dx}{dt}\right|_{t=0} = -\frac{\omega}{\omega^2 + \lambda^2} + \delta$$

(a) Find the general solution of that inhomogeneous ODE. Hint below. (4 points)

(b) Take the general solution of that inhomogeneous ODE, and match it to the initial conditions given. Sketch or plot the solution in an interval that shows the features of interest. (4 points)

(c) Consider the case where $\omega \to \lambda$ for that initial condition. Sketch 2-3 of them, making clear what happens.
What is the limiting case? Find it by considering where above solution goes to. Sketch it. Prove it fulfils ODE. (12 points)

Reminder:
To solve

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{g}(t),$$

one uses the *Variation of constants.*. Using the Ansatz

$$\mathbf{x} = \mathbf{c}(t)e^{\lambda t}$$

and inserting it into the problem yields

$$\dot{\mathbf{x}} = \dot{\mathbf{c}}(t)\mathbf{x} + \mathbf{c}(t)\dot{\mathbf{x}} \tag{1}$$

$$= (\dot{\mathbf{c}}(t) + \lambda\mathbf{c}(t))e^{\lambda t} = A\mathbf{x} + \mathbf{g}(t) = \lambda\mathbf{c}(t)e^{\lambda t} + \mathbf{g}(t), \tag{2}$$

from which

$$\dot{\mathbf{c}}(t)e^{\lambda t} = \mathbf{g}(t)$$

and thus

$$\dot{\mathbf{c}}(t) = \mathbf{g}(t)e^{-\lambda t}$$

follows. One has to integrate this to find $\mathbf{c}$.