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Introduction to Scientific Computing:

Application of Lyapunovs direct method, ODE discretisation
Assignment 9

Exercise 1: *Application of Lyaponvs direct method to a relevant problem* **(10 points)**

Consider the predator-prey system

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y.$$

(a) Remember the linear algebra proofs from the first homeworks and prove that if the matrix A is real and skew-symmetric, then for $\mathbf{x} \in \mathbb{R} < \mathbf{x}, A\mathbf{x} >= \mathbf{0}$ holds. (5 points)

(b) Use Lyaponvs direct method to prove that the nontrivial equilibrium point is stable. Use the square of the systems right hand side as Lyapunov function. (5 points)

Exercise 2: *A one-step method* **(26 points)**

(a) Implement the θ-method, which is given by

$$\mathbf{x}_{n+1} = \mathbf{x}_n + h(\theta \mathbf{f}(x_n) + (1-\theta)\mathbf{f}(\mathbf{x}_{n+1})) \tag{1}$$

(8 points)

(b) Solve

$$\dot{\mathbf{x}} = -\mathbf{x}, \qquad \mathbf{x}(0) = 1 \tag{2}$$

in $[0, 8]$, using stepwith $h = 0.1$ and 2.0, and use $\theta \in \{0, 0.25, 0.5, 0.75, 1\}$. (6 points)

(c) Plot the error for $h = 0.1$ and $\theta \in \{0, 0.25, 0.5, 0.75, 1\}$ (4 points)

(d) Apply the method to the undamped spring mass system (use for example $k/m = 4$ and $x_0 = 2$), using above θ and $h = \alpha\lambda/2\pi$, λ the first eigenvalue of the system, $\alpha = 0.1, 0.05, 0.025, 0.0125, 0.006125$. What do you see? Also make an error plot. (8 points)