

pnqporc

**PORC polynomials for enumerating
groups of order $p^n q$ for $n \leq 5$**

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Contents

1	Enumerating groups of order $p^n q$ for $n \leq 5$	4
1.1	Calculating the generalized PORC polynomials	4
1.2	Evaluating the generalized PORC polynomials	5
	References	6
	Index	7

Chapter 1

Enumerating groups of order $p^n q$ for $n \leq 5$

The package `pnqporc` provides the methods to calculate generalized PORC polynomials in the indeterminates p and q enumerating the number of isomorphism types of groups of order $p^n q$ for $n \leq 5$ and different primes p and q .

These generalized PORC polynomials can contain expressions of the form $d(s, t)$. When evaluating, the value of $d(s, t)$ is 1 if $s \mid t$ and 0 otherwise.

Furthermore, the generalized PORC polynomials can contain expressions of the form $g(p^m - 1, q)$. When evaluating, the value of $g(p^m - 1, q)$ is 1 if m is minimal such that $p^m - 1 \mid q$ and 0 otherwise.

For more details, we refer to the paper [\[EM17\]](#).

1.1 Calculating the generalized PORC polynomials

The following functions compute generalized PORC polynomials enumerating the number of isomorphism types of groups of order $p^n q$ for $n \leq 5$ (or certain subsets).

1.1.1 NumberPNQ

▷ `NumberPNQ(n)` (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of groups of order $p^n q$.

1.1.2 NumberPNQNormalPSylow

▷ `NumberPNQNormalPSylow(n)` (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of non-nilpotent groups of order $p^n q$ with normal Sylow p -subgroup.

1.1.3 NumberPNQNormalQSylow

▷ `NumberPNQNormalQSylow(n)` (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of non-nilpotent groups of order $p^n q$ with normal Sylow q -subgroup.

1.1.4 NumberPNQExceptional

▷ `NumberPNQExceptional(n)` (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of non-nilpotent groups of order $p^n q$, which do not contain a normal Sylow subgroup.

1.1.5 NumberPNQNilpotent

▷ `NumberPNQNilpotent(n)` (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of nilpotent groups of order $p^n q$. This coincides with the number of isomorphism types of groups of order p^n .

1.2 Evaluating the generalized PORC polynomials

1.2.1 EvalPNQPorcPoly

▷ `EvalPNQPorcPoly($poly, p, q$)` (operation)

This evaluates a generalized PORC polynomial returned by one of the functions in the previous section at the given primes p and q .

References

[EM17] B. Eick and T. Moede. *The enumeration of groups of order $p^n q$ for $n \leq 5$* . 2017. [4](#)

Index

EvalPNQPorcPoly, [5](#)

License, [2](#)

NumberPNQ, [4](#)

NumberPNQExceptional, [5](#)

NumberPNQNilpotent, [5](#)

NumberPNQNormalPSylow, [4](#)

NumberPNQNormalQSylow, [4](#)