pnqporc

PORC polynomials for enumerating groups of order $p^n q$ for $n \leq 5$

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Chapter 1

Enumerating groups of order $p^n q$ for $n \leq 5$

The package pnqporc provides the methods to calculate generalized PORC polynomials in the indeterminates p and q enumerating the number of isomorphism types of groups of order $p^n q$ for $n \leq 5$ and different primes p and q.

These generalized PORC polynomials can contain expressions of the form $d(s,t)$. When evaluating, the value of $d(s,t)$ is 1 if $s \mid t$ and 0 otherwise.

Furthermore, the generalized PORC polynomials can contain expressions of the form $g(p^m - 1, q)$. When evaluating, the value of $g(p^m - 1, q)$ is 1 if m is minimal such that $p^m - 1 \mid q$ and 0 otherwise.

For more details, we refer to the paper [EM17].

1.1 Calculating the generalized PORC polynomials

The following functions compute generalized PORC polynomials enumerating the number of isomorphism types of groups of order $p^n q$ for $n \leq 5$ (or certain subsets).

1.1.1 NumberPNQ

▷ NumberPNQ(*n*) (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of groups of order $p^n q$.

1.1.2 NumberPNQNormalPSylow

▷ NumberPNQNormalPSylow(*n*) (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of non-nilpotent groups of order $p^n q$ with normal Sylow p-subgroup.

1.1.3 NumberPNQNormalQSylow

▷ NumberPNQNormalQSylow(*n*) (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of non-nilpotent groups of order $p^n q$ with normal Sylow q-subgroup.

1.1.4 NumberPNQExceptional

▷ NumberPNQExceptional(*n*)   (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of non-nilpotent groups of order $p^n q$, which do not contain a normal Sylow subgroup.

1.1.5 NumberPNQNilpotent

▷ NumberPNQNilpotent(*n*)   (operation)

Given an integer $n \leq 5$, this returns a generalized PORC polynomial in p and q enumerating the number of isomorphism types of nilpotent groups of order $p^n q$. This coincides with the number of isomorphism types of groups of order p^n.

1.2 Evaluating the generalized PORC polynomials

1.2.1 EvalPNQPorcPoly

▷ EvalPNQPorcPoly(*poly, p, q*)   (operation)

This evaluates a generalized PORC polynomial returned by one of the functions in the previous section at the given primes p and q.

References

[EM17] B. Eick and T. Moede. *The enumeration of groups of order $p^n q$ for $n \leq 5$.* 2017. 4

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