

# pnqporc

## PORC polynomials for enumerating groups of order $p^n q$ for $n \leq 5$

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# Chapter 1

## Enumerating groups of order $p^n q$ for $n \leq 5$

The package `pnqporc` provides the methods to calculate generalized PORC polynomials in the indeterminates  $p$  and  $q$  enumerating the number of isomorphism types of groups of order  $p^n q$  for  $n \leq 5$  and different primes  $p$  and  $q$ .

These generalized PORC polynomials can contain expressions of the form  $d(s, t)$ . When evaluating, the value of  $d(s, t)$  is 1 if  $s \mid t$  and 0 otherwise.

Furthermore, the generalized PORC polynomials can contain expressions of the form  $g(p^m - 1, q)$ . When evaluating, the value of  $g(p^m - 1, q)$  is 1 if  $m$  is minimal such that  $p^m - 1 \mid q$  and 0 otherwise.

For more details, we refer to the paper [\[EM17\]](#).

### 1.1 Calculating the generalized PORC polynomials

The following functions compute generalized PORC polynomials enumerating the number of isomorphism types of groups of order  $p^n q$  for  $n \leq 5$  (or certain subsets).

#### 1.1.1 NumberPNQ

▷ `NumberPNQ(n)` (operation)

Given an integer  $n \leq 5$ , this returns a generalized PORC polynomial in  $p$  and  $q$  enumerating the number of isomorphism types of groups of order  $p^n q$ .

#### 1.1.2 NumberPNQNormalPSylow

▷ `NumberPNQNormalPSylow(n)` (operation)

Given an integer  $n \leq 5$ , this returns a generalized PORC polynomial in  $p$  and  $q$  enumerating the number of isomorphism types of non-nilpotent groups of order  $p^n q$  with normal Sylow  $p$ -subgroup.

#### 1.1.3 NumberPNQNormalQSylow

▷ `NumberPNQNormalQSylow(n)` (operation)

Given an integer  $n \leq 5$ , this returns a generalized PORC polynomial in  $p$  and  $q$  enumerating the number of isomorphism types of non-nilpotent groups of order  $p^n q$  with normal Sylow  $q$ -subgroup.

### 1.1.4 NumberPNQExceptional

▷ `NumberPNQExceptional( $n$ )` (operation)

Given an integer  $n \leq 5$ , this returns a generalized PORC polynomial in  $p$  and  $q$  enumerating the number of isomorphism types of non-nilpotent groups of order  $p^n q$ , which do not contain a normal Sylow subgroup.

### 1.1.5 NumberPNQNilpotent

▷ `NumberPNQNilpotent( $n$ )` (operation)

Given an integer  $n \leq 5$ , this returns a generalized PORC polynomial in  $p$  and  $q$  enumerating the number of isomorphism types of nilpotent groups of order  $p^n q$ . This coincides with the number of isomorphism types of groups of order  $p^n$ .

## 1.2 Evaluating the generalized PORC polynomials

### 1.2.1 EvalPNQPorcPoly

▷ `EvalPNQPorcPoly( $poly$ ,  $p$ ,  $q$ )` (operation)

This evaluates a generalized PORC polynomial returned by one of the functions in the previous section at the given primes  $p$  and  $q$ .

# References

[EM17] B. Eick and T. Moede. *The enumeration of groups of order  $p^n q$  for  $n \leq 5$* . 2017. 4

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