Family Name : .... **Exam number :** ....

First name : .... **Matric. number :** ....

Notes on the exam:

Write name and registration number in the corresponding fields. Do **not** use pencils, green or red pens (used in marking). Place name and reg. number on **each sheet**, number sheets **consecutively** and write only on **one side** of the sheets! Memorize or write down the **exam number**.
You are allowed to use a non-programmable pocket calculator and two pages of equations.

Task	1	2	3	4	5	Σ (44)
Mark						

1. Task (10 Points)

Answer briefly the following questions:

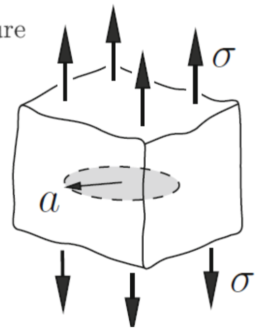
1) What physical assumptions is the Coulomb-Mohr failure criterion based on? For what materials is it appropriate?
2) What is the definition of the material-specific critical crack length? What are approximate values for ductile metals and engineering ceramics?
3) Under which conditions should Irwin's correction be used? What are the underlying assumptions related to the stress field at the crack tip (make a sketch)?
4) Create a diagram of maximum allowable stress versus number of cycles (Wöhler diagram) for materials with and without permament fatigue resistance. Provide an example for each case.
5) Describe qualitatively what process(es) lead to fatigue crack growth. Why is there often a threshold value below which no crack propagation occurs?

2. Task (8 Points)

Creep fracture

A structure with a disc-shaped crack is subjected to a constant tensile stress σ. The material be linear visco-elastic, characterized by the creep function $J(t)$. The crack tip opening displacement (CTOD) immediatelly after the sample is loaded was 1mm. Onset of crack propagation was observed after 10 minutes.

a) Determine the critical crack tip opening displacement.
b) Describe qualitatively how the velocity of crack propagation can be found based on the visco-elastic Dugdale model. Specifically: make a sketch of the model. What are the underlying assumptions. What information are required?
c) Under which conditions/for which materials one has to be especially aware of the possibility of creep crack propagation?
d) Make a sketch of the (normalized) time-to-failure versus inital/critical crack length.



given:

$E_\infty = 3\text{GPa}$, $E_1 = 1.5\text{GPa}$, $\tau_1 = 5\text{min}$

$$\frac{1}{J(t)} = E_\infty + E_1 e^{-\frac{t}{\tau_1}}$$

3. Task (12 Points)

Stress fields and J-integral

A thin disc with a hole is subjected to an internal pressure p.

a) Determine the stress field using the provided *Ansatz* for the Kolosov stress functions Φ, Ψ.

b) Assuming a stress field $[\sigma_r(r,\varphi), \sigma_\varphi(r,\varphi), \tau_{r\varphi}(r,\varphi)]$ as specified below, calculate the J_1-integral along a closed contour at $r = 2r_1$.

given:

$r_2 = 3r_1$, $\nu = 0.5$, E, p,

$\Phi = Az$, $\Psi = \frac{B}{z}$, $z = re^{i\varphi}$, A, B complex

$\sigma_r = -p\left(\frac{r_1}{r}\right)^2$, $\sigma_\varphi = p\left(\frac{r_2}{r}\right)^2$, $\tau_{r\varphi} = 0$

Remark:

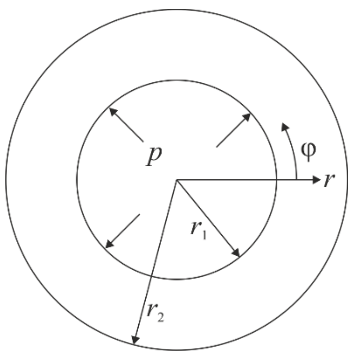
Stress from stress functions:

$\sigma_r + \sigma_\varphi = 2\left(\Phi'(z) + \overline{\Phi'(z)}\right)$

$\sigma_\varphi - \sigma_r + 2\mathrm{i}\tau_{r\varphi} = 2z\left(\Phi''(z) + \Psi'(z)/\bar{z}\right)$

Linear elasticity:

$\boldsymbol{\varepsilon} = \frac{1}{E}\left[(1+\nu)\boldsymbol{\sigma} - \nu \mathrm{tr}\boldsymbol{\sigma}\mathbf{I}\right]$



4. Task (6 Points)

R-resistance

A structure with a center crack under uniaxial tension (geometry factor $Y = 1$) is given. The R-resistance curve of the material be described by a function $R(\delta)$.

a) What are stationary, stable, and instable cracks?

b) What is the condition for stable crack growth?

c) Calculate the range of crack lengths $a_{\min} \le a \le a_{\max}$ for which stable crack growth is possible.

given:

$$R_0, \Delta R, \delta,\ R = R_0\ \{\text{for } a < \delta\}, R = R_0 + \Delta R\left[1 - \left(\frac{\delta}{a}\right)^2\right]\ \{\text{for } a \ge \delta\}$$

5. Task (8 Points)

Stress intensity factor and energy release rate

To determine the critical stress intensity factor and energy release rate a compact tension specimen (CT) as depicted is used. The test is conducted first with a linear-elastic material, then with two specimens of an elasto-plastic material. The forces F, displacements u, and external works W at the onset of crack propagation are given in the table below.

a) For the linear-elastic sample (crack length a_1), determine the critical stress intensity factor K_{IC}.

b) How can the critical energy release rate G_{IC} be determined from measurements of two specimen with different initial crack length? What definition is used? For what materials is this definition valid? At what point in the $F(u)$ curve shall the required quantities be evaluated, and why?

c) For the pair of elasto-plastic samples with initial crack lengths a_2 and a_3 determine G_{IC}.

given:

$t = 2\mathrm{mm}$, $w = 100\mathrm{mm}$, $a_1 = a_2 = 0.4w$, $a_3 = 0.41w$

Remark:

The stress intensity factor for the CT geometry is:

$$K_{\mathrm{I}} \simeq \frac{F}{t}\sqrt{\frac{\pi}{w}}\left[17\left(\frac{a}{w}\right)^{1/2} - 105\left(\frac{a}{w}\right)^{3/2} + 370\left(\frac{a}{w}\right)^{5/2}\right]$$

Sample:	1	2	3
u_{Frac} [mm]	2.5	4	4.5
F_{frac} [N]	1000	500	450
W_{u1} [J]	1250	-	-
W_{u2} [J]	-	800	750
W_{u3} [J]	-	810	770