Theoretical Computer Science 2
Exercise Sheet 6

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Hand in your answers to the Vips directory of the Stud.IP course until wednesday, 04.07.2024 11:59 pm. You should provide your answers either directly as PDF file or as a readable scan or photo of your handwritten notes. Submit your results as a group of four.

Homework Exercise 1: P **completeness [7 points]**
Consider the following problem for contextfree languages.

Emptiness for contextfree languages (ECFL)
Given: A type-2-grammar $G = \langle N, \Sigma, S, P\rangle$.
Question: $\mathcal{L}(G) = \varnothing$?

a) [3 points] Construct a deterministic algorithm, that decides ECFL by requiring not more than polynomial time.
b) [4 points] Show, that ECFL is P-hard wrt. LogSpace-many-one-reductions.

Homework Exercise 2: NP **and graph problems [6 points]**
Examine the following problems with respect to their relation with the class NP.

Vertex Covering (VC)
Given: An undirected graph $G = \langle V, E\rangle$ and $k \in \mathbb{N}$.
Question: Is there a set $S \subseteq V$ with $|S| = k$ and $\forall vEw : v \in S \lor w \in S$?

a) [2 points] Show that VC $\in$ NP holds, by giving a suitable nondeterministic decider, whose time complexity is bounded by some polynome.
b) [4 points] Show that SAT $\leq_m^{\log}$ VC holds by constructing a suitable reduction.
Note: With k and a certain constellation of edges you can enforce, that each vertex covering contains exactly one from each of k vertex pairs.

Homework Exercise 3: PSPACE **and regular languages [7 points]**
Prove Corollary 12.10 from the lecture notes:

a) [2 points] Let $A = \langle Q, \rightarrow, q_0, Q_F\rangle$ be an NFA with $\Sigma^{\leq 2^{|Q|}} \subseteq \mathcal{L}(A)$. Show, that $\mathcal{L}(A) = \Sigma^*$ is valid.
b) [5 points] Prove, that Inclusion and Equivalence for given NFAs are PSPACE-complete wrt. Log-Space many-one reductions. You may use that Universality for NFAs is PSPACE-complete.

Exercise 4:
Show that P is closed under union, concatenation, complement and Kleene operation.

Exercise 5:
Prove that VALIDITY is coNP-complete wrt. logspace-reductions.

VALIDITY	
Given:	Boolean formula φ in CNF.
Question:	Is φ tautological, so that it hold for all assignments?

Exercise 6:
Prove, that ENT is coNP-complete wrt. LogSpace-many-one-reductions.

ENTAILMENT (ENT)	
Given:	Propositional formulas F, F' in conjunctive normal form.
Question:	Does formula F imply F'?

Exercise 7:
Show by constructing an algorithm: If SAT is in P, then we could as well compute a satisfying assignment for each boolean formula in polynomial time.

Exercise 8:
Eine $n^2 \times n^2$ Sudoku-Matrix M ist in n^2 viele $(n \times n)$-Blöcke unterteilt. M ist korrekt ausgefüllt, wenn in jedem Block, in jeder Zeile und in jeder Spalte alle Zahlen von 1 bis n^2 genau einmal vorkommen. Es ist leicht zu sehen, dass SUDOKU in NP liegt, denn wir können die fehlenden Einträge raten und effizient überprüfen. Das heißt auch, dass es eine polytime-Reduktion von SUDOKU auf SAT geben muss.
Finden Sie nun solch eine Reduktion von SUDOKU auf SAT.
Bemerkung: Man kann sogar zeigen, dass SUDOKU NP-vollständig ist.

SUDOKU	
Given:	Eine $n^2 \times n^2$ Sudoku-Matrix M mit Einträgen in $\{1, \dots, n^2, ?\}$
Question:	Gibt es eine Möglichkeit die $?$-Einträge so zu ersetzen, dass ein korrekt ausgefülltes Sudoku herauskommt?

Exercise 9:
Zeigen Sie, dass CLIQUE NP-vollständig bezüglich logspace-many-one-Reduktionen ist.

CLIQUE	
Given:	Ein ungerichteter Graph $G = \langle V, E \rangle$ und eine Zahl $k \in \mathbb{N}$
Question:	Gibt es $S \subseteq V$ mit $\lvert S \rvert = k$ und $\forall u, v \in S\colon \langle u, v \rangle \in E$?